A model for character displacement based on competition between species

References


Genetic drift with polygamy and arbitrary offspring distribution

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Wright (1931) introduced a simple model for a bisexual, diploid population of fixed size and with non-overlapping generations. With $N_1$ males and $N_2$ females, the rate of drift was $\lambda_0 = 1 - (N/8N_1N_2)$ where $N = N_1 + N_2$.

Moran and Watterson (1959) introduced a more general offspring distribution (Wright having assumed a binomial distribution) and treated random mating. They had $N_1$ males and $N_2$ females and a variance of the number of offspring per male of $\sigma_1^2$ and per female $\sigma_2^2$; the means being necessarily $N/N_1$ and $N/N_2$.

For a two allele model, $A$ and $B$ say, Moran and Watterson studied the behaviour of $X_{AB}, X_{AA}, X_{BB}, Y_{AB}, Y_{AA},$ and $Y_{BB}$ the male and female genotype frequencies. By expressing the expectations of these, and of their products, in terms of those of the previous generation they were able to show that the dominant latent root was a root of the equation

$$\lambda^2 - \frac{\lambda}{4N(N - 1)} \left(4N^2 - 2N - \frac{N^3}{N_1N_2} - N_1\sigma_1^2 - N_2\sigma_2^2\right)$$

(1)

for which

$$\lambda_0 \approx 1 - \frac{1}{8N(N - 1)} \left(\frac{N^3}{N_1N_2} - 2N + N_1\sigma_1^2 + N_2^2\sigma_2^2\right).$$

They treated separately a model for random monogamy, for which $N_1 = N_2$ and $\sigma_1^2 = \sigma_2^2$.

In both these models the male and female offspring were assigned to the families at random.

We shall show here how a much more general model can be developed which nonetheless gives rise to exactly the same equation. Instead of assigning offspring to families at random we introduce coefficients $s_{P_{mm}} = \text{prob (two randomly selected males are sibs)}$, $h_{sr}p_{mm} = \text{prob (two randomly selected males are half-sibs through a female)}$, etc., these coefficients encompassing the mating structure.
and being assumed constant through time. We introduce coefficients of identity 
\( \phi_{mm} = \text{prob (two genes randomly selected from two different males in the } g\text{th generation are of the same type)}, \) etc.

One can then obtain equations for \( \lambda_0 \). In the particular case of random assignment of males and females to families we have

\[
\lambda^2 - \lambda (1 - \frac{1}{2} p) + \frac{1}{4} p = 0
\]

where \( p = \phi_{mm} + \frac{1}{2} (\phi_{mf} + \phi_{sm}) \) and is the same if \( mm \) is replaced by \( mf \) or \( ff \), and, moreover, this approach permits a simple interpretation since \( p = \frac{1}{2} E \) (parents in common for two randomly selected individuals), and we can identify \( p \) with \( 4 \times \text{constant term in (1)} \). For \( p \) relatively small

\[
\lambda_0 \approx 1 - \frac{1}{4} p = 1 - \frac{1}{8} E \text{ (parents in common)},
\]

which is precisely the expression appropriate for an equivalent haploid model.

For a model of polygamy in one sex only, with the variance of the number of females (males) per male (female) equal to \( \sigma_f^2 \) or \( \sigma_m^2 \) we have

\[
p = \frac{\sigma_f^2 + 2\sigma_m^2 + 2}{2(N-1)} + o\left(\frac{1}{N^2}\right)
\]

where we have put \( N_1 = N_2 \) for convenience, as compared with

\[
p = \frac{\sigma_f^2 + 2}{2(N-1)}
\]

for monogamy.

Full details of the general model and a variety of special cases will be given in Cannings (1974).

References


The Hardy-Weinberg law with overlapping generations

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The Hardy-Weinberg law is generally regarded as one of the most important results of population genetics. It was originally proved for the case of populations