Kinetic Studies of the Lactic Acid Fermentation in Batch and Continuous Cultures*

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Summary

The fermentation kinetics of the homofermentative organism *Lactobacillus delbrueckii* in a glucose-yeast extract medium is studied in both batch and continuous culture under conditions of controlled pH. From a graphical analysis of the batch data, a mathematical model of the process is derived which relates bacterial growth, glucose utilization, and lactic acid formation. The parameters in the model represent the activity of the organism and are a function of pH, having a maximum value at about 5.90. In a continuous stirred tank fermentor (CSTF), the effect of pH, feed concentration, and residence time is observed. The feed medium is a constant ratio of two parts glucose to one part yeast extract plus added mineral salts. An approximate prediction of the steady-state behavior of the CSTF can be made using a method based on the kinetic model derived for the batch case. In making step changes from one steady state to another, the transient response is observed. Using the kinetic model to simulate the transient period, the calculated behavior qualitatively predicts the observed response.

INTRODUCTION

The application of kinetic models to ordinary chemical reactions has long been regarded as being very important in the design and process control of chemical reactors. However, the development and application of fermentation kinetics has been somewhat slower as a result of the complexity of the process. Nevertheless, a study of the fermentation rates can be very useful for design of both continuous and batch systems and during the last decade a number of models have been proposed for bacterial growth.

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In this work, a study was made of the fermentation kinetics of *Lactobacillus delbrueckii* on a glucose-yeast extract media in both batch and continuous culture under conditions of controlled pH and temperature. For the batch fermentation, a model was developed that relates bacterial production, sugar utilization, and lactic acid production. From this model the steady-state and transient behavior of a continuous, stirred tank fermentor (CSTF) was estimated. The model parameters had a marked dependence on the pH of the fermentation.

**MODEL DEVELOPMENT**

The fermentation was analyzed in terms of a general, irreversible biological reaction,

\[ V + S \rightarrow 2V + P \] (1)

where the bacteria, \( V \), consumes some substrate, \( S \), to produce more bacteria and a product, \( P \). In this study glucose was the primary carbohydrate source for the fermentation and was considered to be the limiting growth component, since the bacterial growth stopped when the glucose concentration reached zero. Yeast extract provided the necessary amino acids and other nutrient requirements. Trace minerals were supplied by adding inorganic salts. Previous studies\(^4\)–\(^6\) have shown that yeast extract has a stimulatory effect on the *L. delbrueckii* fermentation and enhances the bacterial yield. By using a fixed ratio of glucose to yeast extract in the culture medium, changes in the growth kinetics due to yeast extract were minimized. Lactic acid formation represented the fermentation products. While some studies have indicated lactate inhibition of this fermentation,\(^4\)–\(^6\) the concentration of the lactate produced in this study was below the reported limiting concentrations.\(^4\)

Six rate equations commonly used to represent the consumption or formation of components in fermentations were fitted against batch fermentation data.

\[
\frac{dC_i}{dt} = \alpha_i \left( \frac{dC_v}{dt} \right) + \beta_i C_v \\
\frac{dC_i}{dt} = \alpha_i \left( \frac{dC_v}{dt} \right) \\
\frac{dC_i}{dt} = \beta_i C_v \\
\frac{dC_i}{dt} = \beta_i C_v C_s
\] (2–5)

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\[
d\frac{dC_i}{dt} = \alpha_i(dC_i/dt) + \beta_iC_iC_v
\]  
(6)

\[
d\frac{dC_i}{dt} = k_iC_i/(K'_s + C'_s)C_v
\]  
(7)

$C_i$ refers to the concentration of biomass, $C_v$, glucose, $C_s$, or lactate, $C_p$. $\alpha_i$ and $\beta_i$ are appropriate rate constants. In eq. (7) $K'_s$ and $C'_s$ are dimensionless quantities defined as, $K'_s = K_s/C_{o0}$ and $C'_s = C_s/C_{o0}$, where $C_{o0}$ is the initial glucose concentration and $K_s$ is a constant.

Since eqs. (2), (3), and (6) contain a term of $dC_v/dt$ on the right-hand side, they are not applicable to the case of bacterial growth. Equations (2) to (7) could be compared to the batch data in their differential form. However, due to the large error that can occur in graphically differentiating experimental data, an integral method of analysis was used.\(^1\)

**EQUIPMENT AND PROCEDURES**

The techniques used in this study were similar to those employed in previous investigations of fermentations of acid-producing bacteria.\(^4\)\(^,\)\(^7\)\(^-\)\(^9\)

**Apparatus**

A diagram of the apparatus used for the pH controlled fermentation is shown in Figure 1. A 7-liter New Brunswick Fermentor (Model F-07) was used with a New Brunswick Fermentor Drive Assembly (Series FS-300) which provides a temperature controlled water bath and variable speed agitator drive. The pH of the medium was measured with Leeds and Northrup heat-sterilizable electrodes which were connected to a Heathkit pH Recording Electrometer (Model EUW-301) which amplified the signal and drove a recorder pen. When the pH fell below the set point, the recorder pen activated a microswitch which supplied power to an Emdeco Micro Flow Tubing Pump (Model No. 102-130) that accurately metered 4N sodium hydroxide to the fermentor to neutralize the acid formed.

The microswitch also activated a relay which sent a 1-V signal to a strip chart recorder that was run continuously. Thus, when caustic was being added to the fermentor, the trace on the chart showed a hump indicating the time and duration the pump was on. From this record, the rate of acid production was deduced. The set point was adjusted by the zero setting of the recorder.
Agitation of the system was accomplished by two paddle stirrers attached to the drive shaft, one located 2 in. from the bottom and the other 6 in. below the upper liquid surface. The shaft was run at 300 rpm for all batch and continuous fermentations.

When this fermentor was operated continuously, a Cole-Parmer "Masterflex" Variable Speed Tubing Pump (Model No. 7020C) metered the feed medium from a cotton plugged, 5-gal solution bottle to the fermentor, which was maintained at constant volume by an overflow system. The overflow passed through a cold water heat exchanger to cool the culture down to 15°C, which quenched further fermentation. At timed intervals a sample of the overflow was collected in test tubes mounted on a rotary rack in a refrigerated box maintained at 5°C for further analysis.

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Culture and Inoculum Preparation

The homofermentative organism *Lactobacillus delbrueckii* NRRL B-445, obtained from the Northern Utilization Research Branch, United States Department of Agriculture, Peoria, Ill., was used in this study.

Stock cultures were maintained on stabs in a medium composed of 1% glucose, 1% yeast extract, 2% agar, and two marble chips. The cultures were transferred frequently in order to maintain a satisfactory and active inventory. After incubating the stabs for 48 hr at 45°C, a sizable colony was established. Then the stabs were wrapped in polyethylene bags and stored in the refrigerator at 5°C for further use.

Nutrient Sources

The carbohydrate source was A.C.S. Standard Grade anhydrous glucose. Yeast extract (Difco) was used to provide the organic nutrients. Inorganic salts were supplied in excess as described by Gillies.10

The feed medium for the continuous runs was made up in 5-gal bottles and sterilized in a New Brunswick vertical autoclave for 2 hr at 15 psig steam pressure. The fermentor and medium were sterilized for 1 hr.

The feed lines were sterilized prior to use by passing a 1% solution of tetramethylammonium bromide through them, followed by a thorough flushing with a 50% ethanol-water solution. In the process of connecting and disconnecting feed lines from the fermentor, the area around the input ports was thoroughly swabbed with the ethanol solution.

The inoculum for the batch and continuous cultures was prepared in 250-ml shaker flasks from the stab cultures and incubated for 18 hr at 45°C. The medium consisted of 1% yeast extract, 1% glucose, inorganic salts, and powdered calcium carbonate.

Analyses

The bacterial density of the culture was determined from turbidity measurements. Since it was impractical to dilute each sample to the range where the Beer-Lambert Law applies, a correction factor was established that would compensate for such deviations. This factor was determined by measuring the optical density at successive dilu-
tions of a bacterial culture. For optical densities below 0.08, the Beer-Lambert Law was observed to hold. At higher optical densities, the following function was found to apply:

$$\text{OD}_C = (\text{OD}) \exp [0.5745(\text{OD} - 0.08)] \quad \text{OD} > 0.08 \quad (8)$$

where $\text{OD}_C$ is the corrected optical density of the sample and is proportional to the bacterial concentration. To find the bacterial density of the undiluted culture, $C_*$, the number of dilutions, $n$, is multiplied by $\text{OD}_C$ and the optical density of the unfermented medium $\text{OD}_B$, is subtracted from the product.

$$C_* = (n)(\text{OD}_C) - \text{OD}_B \quad (9)$$

The units of $C_*$ are defined as units of optical density per milliliter (UOD/ml).

The optical densities were measured on a Beckman DU Spectrophotometer (Model N. 2400) using light with a wavelength of 6100 Å. The samples were contained in standard silica absorption cells (Beckman No. 75184). Before and after a measurement was made, the instrument calibration was set to 100% transmission using distilled water for a reference.

Glucose was determined using the method of Shaffer and Somogyi, following the procedures outlined by Neish. The lactic acid formation was determined from the record of the caustic addition rate. Also as a check, a number of samples were analyzed for lactate ion by the method of Friedmann and Graesser.

**RESULTS AND DISCUSSION**

*Batch Fermentations*

Batch runs were made at 44.8°C and controlled pH levels of 4.95, 5.33, 5.85, and 6.35. The initial glucose concentration for these runs was about 2% while the yeast extract concentration was about 1%. The bacterial density, glucose concentration, and lactic acid concentration were followed during the course of the fermentation. Samples were withdrawn at approximately one-half hour intervals and immediately analyzed. Using a procedure similar to that employed by Longsworth and MacInnes, corrections were applied to the observed data in order to compensate for the effects of dilution of the media by the neutralizing solution and removal of nutrients during sampling.
The overall lactic acid yield, $\bar{Y}_{P/S}$, defined by,

$$
\bar{Y}_{P/S} = \frac{\text{lactic acid produced (mg/ml)}}{\text{glucose consumed (mg/ml)}}
$$

is compared with the values obtained by Luedeking\(^7\) and Finn\(^15\) in Table I. The maximum yield occurred at a pH of about 5.8. While the findings of Luedeking\(^7\) also show that the yield increases up to pH 5.85, Finn's results indicated that the highest yield occurs at lower values of pH.

<table>
<thead>
<tr>
<th>pH</th>
<th>$\bar{Y}_{P/S}$</th>
<th>$\bar{Y}_{P/S}$</th>
<th>$\bar{Y}_{P/S}$</th>
<th>$\bar{Y}_{P/S}$</th>
<th>$\bar{Y}_{V/S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This work</td>
<td>Luedeking(^a)</td>
<td>Finn(^a)</td>
<td>This work</td>
<td></td>
</tr>
<tr>
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</tr>
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<tr>
<td>6.35</td>
<td>0.82</td>
<td>—</td>
<td>—</td>
<td></td>
<td>0.17</td>
</tr>
</tbody>
</table>

\(^a\) Interpolated

The overall bacterial yield, $\bar{Y}_{V/S}$, as defined in eq. (11) and listed in Table I, also had a maximum at pH 5.8.

$$
\bar{Y}_{V/S} = \frac{\text{bacteria produced (UOD/ml)}}{\text{glucose consumed (mg/ml)}}
$$

In deriving a mathematical model of the batch fermentation, the rate expressions given by eqs. (2) to (7) were integrated and the terms rearranged so that a linear relationship existed between the left- and right-hand sides:

$$
\frac{C_i - C_{i0}}{C_v - C_{v0}} = \alpha_i + \beta_i \left( \frac{\int_0^t C_v dt}{C_v - C_{v0}} \right)
$$

$$
C_i - C_{i0} = \alpha_i (C_v - C_{v0})
$$

$$
C_i - C_{i0} = \beta_i \int_0^t C_v dt
$$
A computer program compared the batch fermentation data with the proposed rate expressions, using the trapezoidal rule for the integration of the experimental data. By using a least squares analysis to compare these integrated expressions with the data, the best fitting rate equations were determined.

The best fit of the bacterial growth data (see Fig. 2) was obtained using eq. (7), which for bacterial growth is the Monod equation. However, the calculated values of $k_v$ and $K_{sv}$ were negative which makes the form of the equation different than the Monod equation.
It was convenient to alter eq. (7) for bacterial growth so that $k_v$ and $K_{sv}$ are positive by changing the denominator from $(K'_{sv} + C_s)$ to $(K'_{sv} - C_s)$:

$$ \frac{dC_v}{dt} = \frac{k_vC_s}{K_{sv} - C_v} $$  \hspace{1cm} (18)

where

$$ K_{sv} = K'_{sv}C_{s0}. \hspace{1cm} (18a) $$

The value of $K_{sv}$ was found to always be greater than the initial glucose concentration, $C_{s0}$, so that the denominator of eq. (18) is never zero or negative. The rate of glucose utilization and lactic acid production were best fitted by forms of eq. (5):

$$ \frac{dC_v}{dt} = -\beta_sC_vC_{s} $$  \hspace{1cm} (19)

$$ \frac{dC_p}{dt} = \beta_pC_vC_{s} $$  \hspace{1cm} (20)

A comparison of eqs. (18) to (20) in the integrated form with the batch data is given in Figure 2 for a fermentation at pH 5.33. From the least squares analysis of the data at each pH level, the values of the parameters $k_v$, $K'_{sv}$, $\beta_s$, and $\beta_p$ were found to be strongly dependent on pH as shown in Figures 3 and 4. The maximum values for the rate constants occurred at about pH 5.9 which confirmed the findings of others who noted a maximum in the acid production and bacterial growth activity of *L. delbrueckii* between pH 5.7 to 6.0. Using the derived model, the actual batch fermentations were simulated on a TR-48 analog computer. Figure 5 shows a comparison of this simulation with the actual data at pH 5.33.

For use in the graphical integration and differentiation of bacterial growth curves, a modified form of the growth rate curve proposed by Edwards and Wilke was applied.

$$ C_v = \frac{M}{1 + \exp \left( \sum_{i=0}^{5} a_it^i \right)} $$  \hspace{1cm} (21)

Rearranging gave,

$$ \ln [(M/C_v) - 1] = \sum_{i=0}^{5} a_it^i $$  \hspace{1cm} (22)

By arbitrarily choosing a value for $M$, the left-hand side of eq. (22) was calculated as a function of time from the experimental data.
Then, using multiple, linear regression analysis, the coefficients, $a_i$, were determined. It was necessary to choose a value of $M$ that was greater than the final value of $C_r$ in order to keep the argument of the natural logarithm from going negative. Choosing a value of $M$ that was 20 per cent higher than the largest value of $C_r$ was adequate for this purpose. Figure 6 shows a fit of eq. (21) with the batch growth data.

**Continuous Fermentations**

The variables investigated during the continuous fermentation were pH, residence time, and feed concentration. The feed composit-
tion always was in the ratio of two parts glucose to one part yeast extract, but the overall concentration was varied. The rate of acid production, concentration of glucose and bacterial density were measured at each steady state and during the transient periods.

The continuous fermentations were started in the same manner as a batch fermentation. When the bacterial density reached the anticipated steady-state level for the continuous fermentation, the flow of nutrients to the fermentor was commenced. The continuous fermentor volume was 5.57 liters. The steady state was determined
Fig. 5. Mathematical simulation of batch fermentation at pH 5.33: (—) model; (O, Δ, □) data.

when the bacterial density, acid production rate, and glucose concentration leveled out, which required from three to six residence times. Samples were taken automatically at 37.5 min intervals so that the transient period between steady states could be observed. It was assumed that the change from one set of operating conditions to the next was instantaneous; however, it actually took about 5 min to establish new conditions of feed rate, pH, and feed concentration.

The operating conditions for each run are listed in Table II with the observed steady-state values of glucose concentration, bacterial density, and lactate concentration. The lactate concentration was determined analytically and also calculated from the rate of hydroxide addition.

To systematically evaluate the effect of the three operating variables on the steady-state operation of the fermentor, the operating conditions were chosen to follow a three-dimensional statistical de-
sign. The conditions for the center point of the design were at pH 5.40, a dilution rate of 0.20 hr⁻¹, and a glucose feed concentration of 20.0 mg/ml. Because of experimental difficulties it was not possible to set the operating variables at exactly the design criteria, but they were reasonably close.

The reproducibility of the center point of the design was not good as shown in Table II by steady states 2, 6, 10, 14, and 18. After steady states were achieved at several different operating levels as shown in Table II, the pH, feed rate, and feed concentration were returned to the center point conditions where the new steady-state activity was observed to be significantly different than previous ones. The reason for this change in bacterial activity could not be determined, although there was observed to be a relationship between the pH of the culture at previous operating levels and this change in activity. When the culture was growing at higher levels of pH, the activity was lower than was previously observed when the base case conditions were repeated, as illustrated by the sequence of runs 2 to 6 and 14 to 18. On the other hand, when the pH was lower during the intervening runs, the activity increased as shown in the sequences 6 to 10 and 14 to 18. This adaptation was not completely reversible.
TABLE II
Steady-State Operating Conditions

<table>
<thead>
<tr>
<th>Steady state</th>
<th>pH</th>
<th>$D_v$, hr$^{-1}$</th>
<th>$C_{u,v}$, mg/ml</th>
<th>$\bar{C}_v$, UOD/ml</th>
<th>$\bar{C}_v$, mg/ml</th>
<th>$R_{p,v}$, min</th>
<th>$\bar{C}_{p,v}$, mg/ml</th>
<th>$\bar{C}_{p,v}$, mg/ml</th>
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<td>1</td>
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<td>0.201</td>
<td>9.8</td>
<td>1.84</td>
<td>3.75</td>
<td>80.0</td>
<td>4.29</td>
<td>3.98</td>
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<td>0.201</td>
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<td>123.0</td>
<td>6.59</td>
<td>5.86</td>
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<td>26.4</td>
<td>3.32</td>
<td>6.10</td>
<td>100.0</td>
<td>14.12</td>
<td>15.58</td>
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<td>14.74</td>
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</tbody>
</table>

* Total acid production rate.

$^{b}$ Determined from caustic addition rate.

$^{c}$ Determined from chemical analysis.

as once the culture was exposed to lower pH levels it tended to maintain the higher level of activity. This phenomenon was also observed in continuous fermentation runs of shorter duration.

The overall yield of lactic acid, $\bar{Y}_{P/S}$, and bacetria, $\bar{Y}_{V/S}$, (Table III) based on sugar consumption did not show the pH dependence observed in the batch fermentations. The average value of the bacterial yield for the continuous fermentation was higher than the average value for the batch fermentation. However, the lactic acid yield was lower. This indicates that the glucose may be utilized more for cell production and less for lactic acid production during the continuous process than in the batch process.

Samples from some of the steady-state continuous cultures were used to inoculate shaker flask cultures. After inoculation the bacterial density was followed for several hours to see if any lag phase.
was observed. In all the cultures studied no discernible lag phase was observed, even for the case of long residence times and zero glucose concentrations in the medium. This implied that the bacteria are in an active state even under rather extreme conditions and that the bacterial density determined from the turbidity measurements was a valid representation of the active biomass. If there had been a lag phase, this would have implied that a significant number of cells were inactive or dead as the turbidity measurement does not distinguish between dead and live cells.

Using the growth model developed for the batch fermentation case given in eqs. (18) to (20) and a general material balance for a CSTF, the unsteady-state equations for continuous fermentations were derived:

\[
\frac{dC_v}{dt} = \left[k_{\text{ov}}C_v/(K_v - C_v)\right]C_v - DC_v \tag{23}
\]

\[
\frac{dC_s}{dt} = (C_{sf} - C_s)D - \beta_s C_s C_v \tag{24}
\]

\[
\frac{dC_p}{dt} = \beta_p C_s C_v - DC_p \tag{25}
\]

where \(D\) is the dilution rate.
By setting the left-hand side of each of the above equations equal to zero, the steady-state solution was determined:

$$\bar{C}_s = DK_{ss}/(k_v + D)$$ (26)

$$\bar{C}_v = [(C_{ss} - \bar{C}_s)/\beta_v\bar{C}_s]D$$ (27)

$$\bar{C}_p = \beta_p\bar{C}_v/D$$ (28)

$K_{ss}$ was found as a function of pH from Figure 4 and eq. (18a). $k_v$, $\beta_v$, and $\beta_p$ were determined from Figures 3 and 4. From the operating conditions (Table II) the values of the steady-state concentrations of each variable were calculated and are listed in Table IV.

**Transient Behavior**

Between each steady state, transient response data was obtained for step changes in the operating conditions. These step changes

<table>
<thead>
<tr>
<th>Steady state</th>
<th>$\bar{C}_{vs}$ UOD/ml</th>
<th>$\bar{C}_v$ mg/ml</th>
<th>$\bar{C}_p$ mg/ml</th>
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involved changes in one, two, or three variables, depending on the requirements of the experimental design. The results for one of these transition periods is plotted in Figure 7, showing the response to a decrease in pH from 5.45 to 4.60. There did not appear to be any oscillatory behavior in the cultures during the transient periods. However, overshoot and undershoot of the final steady state was observed in several of the runs.

Equations (23) to (25) were solved on the analog computer to simulate the transient behavior of the system (Fig. 8). The value of $K_{rev}$ was the same as that used to predict the steady-state behavior. In order that the computed transient run would have the same initial and final values as those observed, the values of $k_v, \beta_s,$ and $\beta_p$ were redefined using eqs. (26) to (28).

$$k_v = \left(\frac{DK_{rev}}{\bar{C}_s} - D\right)$$  \hspace{1cm} (29)

$$\beta_s = \left[\frac{(C_{sf} - \bar{C}_s)}{\bar{C}_s\bar{C}_v}\right]D$$  \hspace{1cm} (30)

$$\beta_p = \frac{D\bar{C}_p}{\bar{C}_s\bar{C}_v}$$  \hspace{1cm} (31)

where $\bar{C}_s, \bar{C}_v,$ and $\bar{C}_p$ were the steady-state values at the end of the transient run.

![Fig. 7. Transient behavior of the continuous fermentation following a step change in the pH.](image-url)
The correspondence between the model and the data was fairly reasonable. The model predicted a much faster response in the glucose concentration than was observed experimentally. The response times for the other variables was approximately the same as observed experimentally.

CONCLUSIONS

The batch and continuous fermentation of *L. delbrueckii* was studied at controlled pH levels in a glucose-yeast extract medium. The bacterial, glucose, and lactic acid concentrations were measured as a function of time. Using a graphical analysis of the batch data, a kinetic model was developed that closely fitted the observed batch data. The parameters in the model, which reflected the activity of the culture, were found to be a function of pH and have maximum values at about pH 5.90.

Using a modified form of the growth curve equation proposed by Edwards and Wilke an adequate mathematical representation of the experimental growth data was obtained. The modification allowed the model to be fitted by a simple linear multiple regression analysis.
Both the overall bacterial and lactic acid yields based on sugar consumption showed a maximum at pH 5.9 for the batch fermentation. In the continuous fermentation, no dependence of the yield on pH was observed.

The effect of pH, feed concentration, and dilution rate on the steady-state behavior of the CSTF was determined by carrying out an experimental program based on a three-dimensional statistical design. In most instances the observed values were approximately predicted by the batch kinetic model.

In moving from one steady state to another, step changes were made in the operating conditions, and the transient behavior of the CSTF was observed in terms of acid production rate, glucose concentration, and bacterial density. Using the batch kinetic model, this transient behavior was simulated on the analog computer. The response time for changes in the glucose concentration was much faster for the simulated runs than was observed, but the response time of the bacterial density was about the same in both cases.

**Nomenclature**

- $C_i$: concentration of component $i$, mg/ml or UOD/ml
- $C_{i0}$: initial concentration of component $i$, mg/ml or UOD/ml
- $C_p$: lactate concentration, mg/ml
- $\tilde{C}_p$: continuous fermentation steady-state lactate concentration, mg/ml
- $C_g$: glucose concentration, mg/ml
- $C_{g0}$: initial glucose concentration in batch fermentation, mg/ml
- $C'_g$: dimensionless glucose concentration ($=\frac{C_g}{C_{g0}}$)
- $C'_g$*: glucose concentration in feed to continuous fermentor, mg/ml
- $\tilde{C}_g$: continuous fermentation steady-state glucose concentration, mg/ml
- $C_b$: bacterial density, UOD/ml
- $C_{b0}$: initial bacterial density in batch fermentation, UOD/ml
- $\tilde{C}_b$: continuous fermentation steady-state bacterial density, UOD/ml
- $D$: dilution rate, hr$^{-1}$
- $K_{ii}$: parameter in rate equation representing component $i$, mg/ml
- $K'_i$: dimensionless parameter ($=\frac{K_{ii}}{C_{i0}}$)
- $K_{ii}$*: parameter in rate equation, mg/ml
- $K'_i*$: dimensionless parameter ($=\frac{K_{ii}}{C_{i0}}$)
- $M$: parameter in growth curve equation, UOD/ml
- $OD$: measured optical density, UOD/ml
- $OD_B$: optical density of unfermented medium, UOD/ml
- $OD_C$: corrected optical density, UOD/ml
- $P$: fermentation product
- $S$: fermentation substrate
- UOD: units of optical density
- $V$: bacteria
\( \bar{Y}_{p,s} \) lactic acid yield based on glucose consumed, dimensionless
\( \bar{Y}_{v,s} \) bacterial yield based on glucose consumed, UOD/mg
\( a_i \) regression parameter in growth curve equation
\( k_i \) rate constant associated with component \( i \), hr\(^{-1}\)
\( k_e \) bacterial growth constant, hr\(^{-1}\)
\( n \) number of dilutions prior to measuring optical density
\( t \) time, hr
\( \alpha_i \) rate parameter associated with component \( i \)
\( \beta_i \) rate parameter associated with component \( i \)
\( \beta_g \) glucose rate constant

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References


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