Prediction of reverse osmosis apparatus performance

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Abstract—The paper derives simple analytical expressions for concentrations of permeate and retentate and for apparatus capacity necessary to reach these, for non-clogging constant property solutions with appreciable osmotic pressure.

INTRODUCTION

The aim of this paper is to show the possibilities of RO and its limitations due to osmotic pressure. For simplicity, the analysis is limited to non-clogging, constant property solutions with osmotic pressure directly proportional to concentration

$$\pi = \beta \cdot c.$$ (1)

For the membrane, we assume linear relationship between permeate velocity and net pressure difference

$$v = A \cdot (\Delta P - \Delta \pi)$$ (2)

where $\Delta P$ is the applied pressure difference across membrane, here assumed constant, and $\Delta \pi$ is the difference between osmotic pressures of the solutions at the high pressure side and the low pressure side of the membrane.

The membrane is further characterized by rejection, assumed constant

$$R = 1 - \frac{c_p}{c_w}$$ (3)

where indices $p$ and $w$ correspond to permeate and high pressure face of the membrane (wall) respectively.

We define concentration polarization modulus

$$M = \frac{c_w}{c}$$ (4)

here $c$ is the local average (mixing cup) concentration.

Our analysis is relevant to all RO processes, where the fraction to be concentrated passes successively through a number of identical modules. It comprises thus both continuous processes with and without recirculation and batch processes. The simplicity of the solutions to be derived further on is caused by the fact, that we assume it possible to replace the local value of the concentration polarization modulus by an average value.

The CP modulus is a complex function of material properties and flow parameters. In a real process, as the pressurized solution gets more and more concentrated, the permeate flow decreases due to osmotic pressure, decreasing in its turn the CP. On the other hand, the axial velocity of the pressurized solution decreases, thus increasing the CP. In a well designed system (and the manufacturers certainly claim to market only such ones), the absolute value of CP is not high. It follows, that, even the variations in CP can not be high.

Brian[1] has shown, that a solution of the CP equation at constant flux is a good approximation to the exact solution, even in a relatively long channel, where the permeate flux drops to 20 per cent of its initial value as a result of the concentration polarization build-up. In a real system, the mixing events following each of the relatively short channels interfere with excessive concentration build-up, so the over-all variation should not be so high. Thus we feel confident to use an average value of CP in our calculations. Following Brian, we suggest to calculate $M$ for a point
halfway down one of the modules, and assume
that the axial flow velocity corresponds in this
point to the average between feed and retentate
(concentrate) flow, and the permeate velocity is
equal to the total permeate flow divided by
apparatus area, and use this $M$ as the average
value of $M$ for the whole apparatus.

For mathematical convenience, we replace the
system in Fig. 1 by a continuous system as in
Fig. 2 and formulate at a point $x$ the liquid
balance

$$dQ = -s \cdot v \cdot dx$$

and the salt balance

$$d(Q \cdot c) = -c_p \cdot s \cdot v \cdot dx = c_p \cdot dQ$$

Eqs. (5, 1, 2) yield

$$dQ = -s \cdot A(\Delta P - \beta(c_w - c_p))dx$$

and with Eqs. (3, 4)

$$dQ = -s \cdot A(\Delta P - \beta RMc)dx.$$  

Let us introduce dimensionless variables

$$q = \frac{Q}{Q_{in}} \quad C = \frac{c}{c_{in}} \quad xi = \frac{x}{l} \quad (9)$$

and constants

$$B = \frac{\beta c_{in}}{\Delta P}$$

ratio of osmotic pressure of feed to applied
pressure difference

$$\Delta_0 = \frac{s \cdot l \cdot A \cdot \Delta P}{Q_{in}}$$

'pure water recovery ratio' or 'dimensionless
apparatus capacity', the ratio of permeate to feed
if we were working with pure water.

Equation (8) may now be written

$$dq = -A_0(1 - MBRC) \frac{d\xi}{d\xi}$$

and Eq. (6)

$$d(q \cdot C) = dq(1 - R) \cdot C \cdot M.$$  

Equations (12) and (13) give

$$q \frac{dq}{d\xi} = \frac{dq}{d\xi} \left( \frac{dq}{d\xi} + \Delta_0 \right) \left[ (1 - R) M - 1 \right]$$

with boundary conditions

$$\xi = 0 \quad q = 1$$

$$\frac{dq}{d\xi} = -\Delta_0(1 - MBR).$$

Equation (14) may be integrated once to give

$$\frac{dq}{d\xi} = -\Delta_0[1 - MBRq^{(1 - R)M - 1}].$$

Comparing Eqs. (12) and (16) the local mixing
cup concentration $C$ is

$$C = q^{(1 - R)M - 1}$$

as a function of the 'remaining bulk flow' $q$. 

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In many applications, it is convenient to define the ratio of permeate to feed,
\[ \Delta = \frac{\int sA (\Delta P - \Delta \pi) \, dx}{Q_{in}} = 1 - q \]  
(18)
the so-called ‘permeate recovery’.
It is easily shown that the average total concentration of permeate is
\[ C_p = \frac{1 - q^{(1 - R)M}}{\Delta}. \]  
(19)

The Eqs. (19) and (17) and Fig. 3 should be of some help in assessing the possibilities of RO application. The term \((1 - R)M\) in the exponent is best interpreted as a characteristic of the membrane under given operating conditions giving the ratio of permeate to feed concentrations in a test cell with the same concentration polarization. The term \(1 - (1 - R)M\) is thus the apparent rejection.

Equations (17) and (19) offer no information at all about equipment dimensions for the RO process. This information may be gained by integrating Eq. (16). Here it should be recalled, that \(dq/d\xi\) is a measure of the local permeate flow,

in Eq. (16) expressed not as usual, as a function of longitudinal distance, but as a function of the ‘remaining bulk flow’ \(q\) or of the recovered permeate \(\Delta = 1 - q\).

The term MBR may be considered a single parameter, accounting for the effect of apparent osmotic pressure.

A plot of Eq. (16), Fig. 4, shows that for an osmotically active feed, it is impossible to recover more than a certain amount of permeate, \(\Delta_{max}\),
\[ \Delta_{max} = 1 - (MBR)\frac{1}{1 - (1 - R)M}. \]  
(20)

Further it shows that the last few per cent of recovered permeate are very costly in terms of membrane area, as the local permeate flow rate falls steeply in proximity of \(\Delta_{max}\).

It is impossible to integrate Eq. (18) for a closed analytical expression.

We have found that under certain conditions \((0 \leq MBR < 0.5, 0 < (1 - R)M < 0.4, \text{ lowest permeate flow} > 5 \text{ per cent pure water flow})\)
\[ \Delta_0 = \frac{s \cdot J \cdot A \cdot \Delta P}{Q_{in}} \]
\[ = 1 - q^{RA - MBR} \ln \frac{q^{RA - MBR}}{1 - MBR} + \frac{q^{RA - q}}{1 - MBR} \]  
(21)
is an approximation with a maximum error less than 5 per cent.

Thus for a requested permeate recovery or, via Eqs. (17) or (19), a requested degree of concentration or permeate quality Eq. (21) delivers directly the necessary membrane area.

Figure 5 is a plot of $\Delta_0/\Delta$ from Eq. (21). It shows the increase of nominal apparatus capacity necessary to reach the same permeate recovery with osmotically active feed as compared to pure water.

It remains to be shown that our simplified approach is valid. We have performed a series of numerical computations channel by channel, for the DDS plate and frame apparatus, consisting of 100 membrane channels in series[2], assuming a NaCl solution as feed and a wide range of operating conditions and compared the results to our simplified predictions.

Equation (21) holds 5 per cent good for recoveries up to at least 85 per cent of the maximum permeate recovery $\Delta_{\text{max}}$, and permeate rates up to 100 g/ft²d.

Equation (19), the permeate concentration relationship, has in the same conditions a maximum error of less than 10 per cent when worst. However, 10 per cent change in permeate concentration requires just a 0.5 per cent deviation in rejection at $R = 95$ per cent and we feel that in manufacture, much higher variations are the rule.

Acknowledgement—This work has been supported by a grant from the Swedish Board for Technical Development.

**NOTATION**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>membrane permeability constant</td>
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<tr>
<td>$B$</td>
<td>ratio of osmotic pressure of feed to the applied pressure difference $B = \beta c_{\text{in}}/\Delta P$</td>
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<tr>
<td>$c$</td>
<td>concentration</td>
</tr>
<tr>
<td>$C$</td>
<td>dimensionless concentration</td>
</tr>
<tr>
<td>$l$</td>
<td>total length of all channels</td>
</tr>
<tr>
<td>$M$</td>
<td>concentration polarization modulus $M = c_{\text{in}}/c$</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>applied pressure difference across membrane</td>
</tr>
<tr>
<td>$Q$</td>
<td>volume bulk flow</td>
</tr>
<tr>
<td>$q$</td>
<td>dimensionless flow $q = (Q/Q_{\text{in}})$</td>
</tr>
<tr>
<td>$R$</td>
<td>solute rejection</td>
</tr>
<tr>
<td>$R_A$</td>
<td>apparent solute rejection $R_A = 1 - (1 - R)M$</td>
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<tr>
<td>$s$</td>
<td>channel width</td>
</tr>
<tr>
<td>$v$</td>
<td>permeate velocity</td>
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<tr>
<td>$x$</td>
<td>an axial dimension parameter</td>
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**Greek symbols**

<table>
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<th>Symbol</th>
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<tr>
<td>$\beta$</td>
<td>osmotic coefficient</td>
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<tr>
<td>$\Delta_0$</td>
<td>pure water recovery ratio, dimensionless apparatus capacity $\Delta_0 = (s \cdot R \cdot A \cdot \Delta P)/Q_{\text{in}}$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>permeate recovery $\Delta = 1 - q$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>osmotic pressure</td>
</tr>
<tr>
<td>$\xi$</td>
<td>dimensionless axial parameter $\xi = (x/l)$</td>
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**Indices**

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<tr>
<th>Index</th>
<th>Description</th>
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<tbody>
<tr>
<td>$p$</td>
<td>permeate, low pressure face of the membrane</td>
</tr>
<tr>
<td>$w$</td>
<td>wall, high pressure face of the membrane</td>
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</table>

**REFERENCES**

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Résumé—Les auteurs dérivent des expressions analytiques simples pour les concentrations de corps qui ont traversé la membrane et de ceux qui ont été retenus et pour la capacité nécessaire de l’appareil à obtenir ces concentrations, dans le cas de solutions ayant des propriétés constantes, la faculté de non-colmatage et une pression osmotique importante.

Zusammenfassung—In diesem Artikel werden einfache analytische Ausdrücke abgeleitet für die Konzentrationen von durchgelassenem und zurückgehaltenem Material und für die zur Erreichung dieser Konzentrationen erforderliche Apparatgröße, für nichtverstopfende, konstante Lösungen mit beträchtlichem osmotischem Druck.