Computer Simulation of the Snowmelt and Soil Thermal Regime at Barrow, Alaska

S. I. Outcalt, C. Goodwin, G. Weller, and J. Brown

An annual snow-soil simulator for arctic tundra was developed by using coupled models of surface equilibrium temperature and substrate thermal diffusion. Snow ripening, melt, and accumulation are modeled in the simulator which is forced with daily weather data. The simulator predicts that a snow fence array capable of producing drift deeper than 4.2 m will initiate a permanent snowfield at Barrow, Alaska. Such a man-induced snowfield could serve as a reliable source of freshwater for Barrow and similar villages in the north slope region of Alaska. Further analysis indicated that albedo reduction due to dust fall, snow removal, etc., is dominant over aerodynamic effects in producing the early spring meltout observed at Barrow Village.

The annual snow and soil thermal regime at Barrow, Alaska, has been the subject of an intensive research effort during the past two decades. That effort has been accelerated during the last 5 yr as a result of the selection of Barrow as the major United States International Biological Program Tundra Biome study site. Detailed energy budget climatological data are available for this site on the Alaska coastal plain [e.g., Weller, et al., 1972; Nakano and Brown, 1972; Maykut and Church, 1973; Brown et al., 1968]. Paralleling this research, the Cold Regions Research and Engineering Laboratory (CRREL) began experimenting with snow fences to augment snow accumulation in the small watershed used by Barrow Village for its municipal water supply [Slaughter et al., 1975]. These experiments posed an interesting series of questions concerning the relationship between the natural, annual snow hydrology and the soil temperature of the coastal tundra, as these regimes are disturbed in either an inadvertent or planned manner during the process of urbanization.

During the annual cycle at an undisturbed flat tundra site the winter snow has accumulated to a depth of 40-50 cm by late May. In early June the snowpack ripens, becoming isothermal at 0°C. This is followed by rapid melt, the bulk of the annual runoff being generated as meltwater in a 3- to 5-day period, leaving the terrain nearly snow-free by mid-June. The significance of this concentration of annual runoff in a short time period can hardly be overestimated. During the premelt period the snow surface has a relatively high albedo (0.87) and is aerodynamically smooth, the roughness length being estimated to be of the order of millimeters. Within this prerunoff period, cloudy days are important in the snow-ripening process. Thermal radiation from the cloud base is a significant heat source for the ripening process when the cloud base temperatures are above freezing. As the snow cover cannot come to thermal equilibrium above the ice point, low, warm, continuous stratus cloud cover promotes rapid ripening when the solar albedo is still relatively high. Meltwater generated in the near-surface layers penetrates to the cold snow at depth releasing the heat of fusion upon refreezing and thus warming that zone toward the ice point. On windy days when the air temperature is above freezing, the sensible heat flux is a significant heat source. Condensation provides additional heat for generating meltwater at the surface.

When the pack becomes isothermal and runoff begins, there is a systematic increase in the roughness length and a decrease in the solar albedo as snow depth decreases. The snow-free tundra has a solar albedo of 0.17 and an aerodynamic roughness length of the order of 1-2 cm. The mechanisms which promote these changes are both local and regional. The increase in snow-free area lowers the terrain albedo while exposing microtopographic features which increase the surface roughness. On a local (small area) scale, scale, dust and organic debris blown from snow-free terrain and the appearance of plant stems above the snow surface reduce the local albedo. Solar radiation increasingly becomes more significant as a heat source due to these albedo reductions. Similarly, the magnitude of turbulent transfer is increased as the melting snow exposes the rough underlying surface composed of microrelief features (polygon centers, rims, and troughs).

When an area has become snow-free, the vegetated surface reaches temperatures higher than the ice point, and the ice-rich organic layer (2- to 15-cm mean value range) begins to thaw. During the course of the summer the thaw zone (active layer) penetrates into the underlying mineral soil layers reaching a maximum depth in silts of from 30 to 40 cm by late August. When autumn arrives, the active layer refreezes from both above and below [Brewer, 1958; Mackay, 1973], and as the winter snow arrives, the accumulation-melt cycle begins anew.

Man can disturb this natural system in several ways. Snow fencing is a planned disturbance that increases snow depth behind a synthetic obstruction to air flow. At such a site the total snow depth is increased, and the total runoff can be augmented during an extended melt-runoff period. Experiments with a 2.7-m fence during the 1972-1973 water year increased the snowdepth by five fold and delayed meltout by 1 month [Slaughter et al., 1975]. As the soil surface cannot attain temperatures greater than 0°C with an overlying snow cover, this effect must induce a disturbance in the evolution of the thaw zone compared to the area outside the influence of the fence. If fencing were carried out on a scale which would eliminate edge effects in the center of the drift area, one would anticipate a decrease in summer substrate temperatures year after year until a new equilibrium with the fenced drift environment was reached.

The snow fence experiment is an example of a planned modification, but what of inadvertent or unplanned effects on the hydrologic and thermal regimes? All construction in the

1 Department of Geography, University of Michigan, Ann Arbor, Michigan 48104.
2 Geophysical Institute, University of Alaska, Fairbanks, Alaska 99701.
3 U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire 03755.

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North modifies to some degree the thermal, radiative, and aerodynamic properties of the terrain. Road, dam, levee, and airfield construction radically change the thermal resistance of the surface layers, while buildings and other above-ground structures also modify the aerodynamic roughness to an extreme degree while producing snowdrifts. An urban area in the flat coastal tundra must have a lowered albedo and an increased roughness length compared with the terrain beyond its margins. It has been noted that snow disappears from Barrow Village approximately 2 weeks before this event occurs on the open tundra during the annual meltout (R. Lewellen, personal communication, 1974).

If it were possible to develop a simulator for this interactive system in which weather, tundra surface, and human modification are the main components, that model could be used to estimate the effects of snow depth modifications and to perform a sensitivity test of hypotheses concerning early meltout in urban areas. Once such a model was considered reliable it could be used to design structures with radiative, thermal, and aerodynamic properties which would both ensure the stability of the structures themselves and attenuate undesirable environmental impact beyond the construction area.

Lachenbruch [1959] made a significant start in this direction utilizing analytical solutions to the periodic heat flow problem. However, these analytical solutions do not incorporate aerodynamic and radiative effects. Analytical formulations become cumbersome when more than two layers are considered and when they require a surface thermal regime to force the substrate thermal evolution. Unfortunately, the surface thermal regime is part of the answer to the terrain modification problem and not part of the question. A model structure is required which can simulate the evolution of the thermal structure of substrates composed of three or more layers which is forced by a surface thermal regime modeled as a time-dependent function of local weather and the physical properties of the site. A surface thermal model of this type was developed by Myrup [1969] for the synthetic analysis of the urban heat island effect on an analog computer. That diurnal surface equilibrium temperature model was modified for use on a digital computer and coupled to a finite difference soil thermal model by Outcalt [1972]. This model was modified to drive an annual snow soil simulator, which will be described in the next section of this paper.

**Overview of the Model Structure and Operation**

The model operates starting from an initial thermal profile which specifies temperature as a function of depth at some starting time and is then forced by daily weather data. For this study, information from the U.S. National Weather Service at Barrow Village was used. The daily meteorological variables used to force the annual simulator are listed in Table 1.

**TABLE 1. Daily Weather Data**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME</td>
<td>month, day</td>
<td></td>
</tr>
</tbody>
</table>
| SUN      | mean daily downward solar radiation | mL/m
| CLO      | cloud cover fraction (0.0-1) |        |
| TA       | mean daily air temperature | °C    |
| RHA      | mean daily relative humidity fraction | RH/100 |
| UA       | mean daily wind speed | cm/s   |
| P        | station pressure | mbars |
| SF       | snowfall | cm    |

The main program first acquires the initial temperature profile, read by subroutine TSTART. Then subroutine FOMO estimates the values of the Fourier modulus (see (1a)) for snow, and organic and mineral soil. These values are later corrected for temperature. The entire matrix of daily weather data is read by using subroutine REAWEA.

After this preparation, daily calculations are begun by sequential calls on subroutine SEARCH, which estimates the equilibrium temperature of the surface, GTEMP, which updates the temperature depth profile, and TUNPIC, which displays the current thermal distribution using line printer graphic techniques. The model flow chart is presented as Figure 1. Each of these subroutines will be described in detail.

**Operation of Subroutines**

**TSTART.** This routine reads in the initial thermal profile data.

**FOMO.** This routine first sets the thermal diffusivities for the three materials used in the model (snow and organic and mineral soils). These values are later corrected for temperature to simulate soil diffusivities during freeze-thaw and in the frozen state. The routine also sets up the spacing between the thermal computation nodes in the substrate. Currently a 5-cm spacing is employed throughout the snow levels down to a depth of 145 cm in the soil. Beyond that depth the node spacing is increased by a factor of 1.25 at each new depth. This places the last node (node 150) well below the depth of annual thermal fluctuation. The surface is located at node 100, and the organic layer extends to node 103 simulating an organic layer 15 cm deep. The thermal properties are initialized at these levels, and Fourier modulus values are computed for each level. The Fourier modulus $F$ is a dimensionless number used in the calculation of substrate thermal evolution (see (1a) and the notation list).

$$F_i = \frac{[D(Z, T_i^{-1})Δt]/(ΔZ^2)}{(1a)}$$

In the simulations described in this paper a base computation diffusivity of $3 \times 10^{-4}$ cm² s⁻¹ was used for mineral soil and cold snow, and a value of $1 \times 10^{-3}$ cm² s⁻¹ was assigned to the organic layer. The Fourier modulus is used in transforming the thermal diffusion equation into a finite difference form that
is actually a smoothing filter in which the weights are prescribed by the value of the Fourier modulus. These relationships are expressed in equations (1b)-(1e).

\[ \frac{\partial T}{\partial t} = D(Z, T) \frac{\partial^4 T}{\partial Z^4} \]  
(1b)

Define reverse time \((r^*)\), then

\[ \frac{\partial r^*}{\partial t} = -1 \]  
(1c)

Forward step (explicit scheme)

\[ T_{i+1} = T_i + (1 - 2F_i)T_{i-1} \]  
(1d)

Backward step (explicit scheme)

\[ T_{i+1} = -F_i(T_{i+1} - T_{i-1}) + (1 + 2F_i)T_{i+1} \]  
(1e)

These relationships are employed in explicit solutions \((1d)) in which the Fourier modulus is constrained to values less than 1/2 to insure numerical stability and in the construction of a tridiagonal matrix of future temperatures \((1e)) employed in implicit solutions [Carnahan et al., 1969]. In the computer simulations an implicit scheme was used with a diurnal time step.

**REAWEA.** This routine reads in weather data and performs the computations necessary to create intermediate variables used in the equilibrium temperature search routine. These operations are summarized in equation (2) (see notation list and Table 1).

\[ Q = f(TAK, RHA, P) \]  
(2a)

\[ RSKY = BB(TSKY)*(1. - CLO**2) \]  
(2b)

For \((TAK \geq 275.)\),

\[ ZO = ZOT \]  
(2c)

For \((TAK < 275. \text{ and } TAK > 271.)\),

\[ ZO = (ZOS + ZOT)/2 \]  
(2d)

\[ EXCO = (XKAR2*AIRDEN*UA) \]  
(2e)

\[ \div (ALOG(ZA/ZO)**2.) \]  
(2f)

**SEARCH.** This subroutine carries out a search for the equilibrium temperature of the surface. An accounting of the surface position is maintained at each iteration by using equation (3a):

\[ SD = SD - ABLA + SF*SDFA \]  
(3a)

The albedo of the surface was fitted to field data and snow depth measurements gathered by Weller. Thus the albedo was calculated at each iteration in the manner described by equations \((3b)-(3g)).

For \((SD,GE,GT,0.)\),

\[ ALB = ALBT + .046*SD \]  
(3e)

For \((SD,LE,0.)\),

\[ ALB = ALBT \]  
(3f)

For \((SD,GE,SDMAX,AND,TAK,LT,268.)\),

\[ ALB = ALBS \]  
(3g)

Net solar radiation was calculated by using equation (3h):

\[ RSN = SUN*(1. - ALB) \]  
(3i)

At this point the program enters the equilibrium temperature search loop and uses equations \((3i)-(3n)) in the computation of the values of the energy budget components.

\[ RLN = RSKY - BB(TEQ) \]  
(3i)

\[ RN = RSN + RLN \]  
(3j)

\[ S = (C/DZ)*(T(LEQ+1) - TEQ) \]  
(3k)

\[ H = EXCO*AIRCAP*(TAK - TEQ) \]  
(3l)

\[ QG = f(TEQ,SRHF, P) \]  
(3m)

\[ LE = EXCO*LHEAT*(Q - QG) \]  
(3n)

Note that each of the surface energy budget components in equations \((3i)-(3n)) \((RN, S, H, LE)) is a function of surface temperature \((TEQ)) only when the equations have been specified in this form and all other information is known. The familiar energy conservation equation can then be stated as a function of surface temperature in equation \((3o)).

\[ RN + S + H + LE = BAL(TEQ) \]  
(3o)

Note that if BAL(TEQ), the sum of the energy budget components, is sufficiently close to zero, all the energy budget components are correctly specified, and the value of the surface temperature is also correct. In practice, this region is \(\pm 1 \text{ Ly/d.}\) The solution for a sufficiently small BAL(TEQ) is accomplished by making two initial guesses at the surface temperature using an interval halving algorithm followed by further estimates for solution surface temperatures \((TEQ)) using the secant algorithm [Beckett and Hurt, 1967]. This scheme yields convergence within the prescribed accuracy \((\pm 1 \text{ Ly/d}) with less than 8 iterations.

If snow is present and the surface equilibrium temperature is above 0°C, it is obvious that energy is available to either ripen or melt snow cover. In this event the surface equilibrium temperature is reset to the ice point, and the substrate soil temperature nodes are inventoried to establish if all nodes representing snow cover are at the ice point. The thermal evolution of the nodes representing snow cover is constrained to temperatures at or below the ice point. The energy budget components are then recalculated with the surface equilibrium temperature at 0°C, and the heat flux from the top layer to the snow surface is set to zero. The sum of the recalculated energy budget components represents the energy available to either warm or melt snow. In all computations, heat flow toward the surface from either above or below is considered positive. If cold snow is present, the available heat is distributed downward through the snowpack, removing the cold content from the surface downward.

This scheme attempts to simulate the near-surface melt and refreezing of meltwater at depth which warms the pack to the ice point. An algorithm for the simulation of density evolution
within the snowpack has not been constructed. Bulk density is currently being used to estimate density-dependent thermal properties. At this stage, only the total water content of the snowpack is considered. Thus discrepancies with field data can be expected when ablation occurs in snow with strong vertical density gradients. This routine is exited, furnishing a new surface temperature (daily mean) and diurnal totals for the energy budget components.

**GTEMP.** This subroutine sets all temperatures at and above the current surface level to the equilibrium temperature solution furnished by SEARCH. This procedure is useful in the event of new snowfall. An additional subroutine is called which adjusts the Fourier modulus of each layer for the temperature of that layer at the last iteration. In this manner the nonlinear parabolic partial differential equation representing the diffusion of temperature in the substrate (equation (1b)) is made linear at each diurnal iteration when calculations are carried out in a finite difference mode (equation (1)). The routine used in this operation employs functions which synthesize the temperature-dependent behavior of thermal diffusivity through the ice point. The scheme is abstracted by the expressions in equation (4).

\[
TC = \frac{1}{2}(T(L-1)+T(L+1))+\frac{1}{2}T(L) \quad (4a)
\]

\[
\text{XER} = \text{ERFC}(\text{ABS}(TC/1.8)^{\text{FHEAT}}) \quad (4b)
\]

\[
\text{FOMOD}(L) = \text{FO}(L)^*(C(ML) \div (C(ML)+\text{XER}^*\text{POR}(ML))) \quad (4c)
\]

For \((TC.LT.-7.)\),

\[
\text{FOMOD}(L) = \text{FO}(L)/(1.-\text{POR}(ML)^*0.5)) \quad (4d)
\]

For \((TC.GT.0.0)\),

\[
\text{FOMOD}(L) = \text{FO}(L) \quad (4e)
\]

This scheme is extremely generalized, as there is considerable uncertainty about the spatial variance of ice content in the soil and the thermal properties of the organic layer. Thus we have constructed a soil sequence with thermal properties which synthesize the temperature-dependent response of real soils. In addition, an option is available to modify the relative humidity of the soil surface as a function of active layer depth to simulate desiccation effects due to drainage and/or evaporation during the summer season.

Next a routine is called which solves the simultaneous equations which are represented by a tridiagonal matrix with columns represented by the right side of equation (1e) and a thermal vector at the last time step represented by the left side of equation (1e). This implicit system of equations is solved by a recursive scheme [Carnahan et al., 1969]. The solution is checked to ensure that no snow nodes are above 0°C. The temperature at the lowest node is never changed and remains fixed at the value set by \(T_{\text{START}}\). The temperatures at the surface and above are continually set at the equilibrium temperature by subroutine SEARCH.

**TUNPIC.** This subroutine uses line printer graphic techniques to portray the soil-snow thermal structure and the snow depth throughout time on a daily basis. Each line printer symbol represents a depth node times 1 day.

**DEVELOPMENT HISTORY OF THIS SIMULATOR**

The annual modeling effort was initiated by the development of a simple active layer simulator at the University of Michigan [Goodwin, 1972]. This model, forced by mean monthly weather data, utilized many of the subroutines designed for diurnal simulation [Outcalt, 1972] and calculated the active layer depth from energy conservation considerations and the temperature profile in the neighborhood of the soil surface. Later, a soil thermal diffusion routine was incorporated into a model which was forced with daily weather information in a program flow structure similar to that of the present model, the major differences being the use of an explicit finite difference formulation of the soil thermal evolution equation (see equation (1d)) and the use of the interval-halving algorithm in the search for the surface equilibrium temperature. The development of that model has been reported elsewhere [Goodwin and Outcalt, 1975]. The performance of the model was evaluated in a detailed comparison with field measurements of the energy transfer regime and substrate thermal structure [Outcalt et al., 1975].

The analysis demonstrated that the time dependence of meltout and total soil thaw were extremely close to the field measurements (2 days, 10 cm). However, total daily heat flux components diverged from the field-measured values. Further, a study of the surface and substrate temperatures indicated that a major source of this error was traceable to the empirical estimation of the thermal radiation flux from the sky hemisphere. There is also considerable uncertainty as to the thermal properties of snow and the organic layer during the annual weather cycle, traceable to freeze-thaw and desiccation effects. A considerable improvement in simulator accuracy is to be anticipated with the completion of a coupled soil temperature water flow model now under development by the senior author.

The current simulator was produced by constructing an implicit scheme for soil thermal evolution and adding the secant algorithm to the equilibrium temperature search routine. These modifications increased greatly program efficiency, reducing computation time by nearly an order of magnitude. It should be mentioned that when the implicit scheme was used during snow ripening, it was necessary to decouple the snow from the underlying soil to prevent the cold from moving upward into the pack. This is accomplished by setting the two lowest nodes in the snow at the ice point when the surface is at the ice point. This may be considered to represent the low-density basal layer and/or meltwater flow along the snow-soil interface [Colbeck, 1974].

The simulation cannot be improved until a set of field data including total downward radiation becomes available. At that stage it should be possible to force the simulator with both incoming solar and thermal radiation. Then attention can be directed toward the development of a simulator with a coupled heat-water flow algorithm, since that sensitivity is now masked by thermal radiation estimation errors as they are incorporated into equilibrium temperature solutions at the surface. Therefore this paper considers only the gross time-dependent response of snowmelt and summer soil thaw evolution.

**TABLE 2. Time Dependence Comparison, Undisturbed Tundra**

<table>
<thead>
<tr>
<th>Event</th>
<th>Field</th>
<th>Simulation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melt begins</td>
<td>June 4</td>
<td>June 5</td>
</tr>
<tr>
<td>Melt complete</td>
<td>June 13</td>
<td>June 14</td>
</tr>
<tr>
<td>Maximum thaw depth (cm)</td>
<td>35</td>
<td>30+†</td>
</tr>
</tbody>
</table>

* Slight soil desiccation simulated.
† Resolution only 5 cm in graphic output.
SIMULATION OF SNOW FENCE EFFECTS

The simulator was run using a chain of weather data beginning on February 10, 1971, at which time a field-measured snow-soil thermal profile was available [Weller and Holmgren, 1974]. The comparison of simulated and field events at the undisturbed tundra site appears in Table 2 [Weller et al., 1972]. A sample of the type of line printer graphical output used in the analysis is presented in Figure 2.

The snow depth was gradually increased by scaling the snow depth factor (SDFAC equation (3a)) upward from unity. The snow fence simulation was run by applying this increased factor after the simulator ran for a year in the undisturbed mode (SDFAC = 1). The simulator was forced in the fence simulation by replaying the 1971 weather data repetitively for up to 4yr. In the simulation of a snow fence 2.7 m high, meltout was delayed until July 19. This corresponds roughly with field observations in 1973 when meltout was delayed until July 18 behind a fenced drift which was 2.7 m deep at the onset of the melt season. This correspondence must be viewed as being fortuitous, since 1971 weather was used, not the 1973 weather! Figure 3 is an aerial oblique of the actual snow fence. The 2.7-m drift is seen in the upper section.

The monthly mean temperatures for the period 1945-1971 have been arrayed in standard deviation form as departures from the mean monthly means. The standard deviation values for June and July 1971 were +1.3 and +0.9. Thus the '1971 simulation weather' during the melt season was considerably warmer than the 1945-1971 norm. With these constraints in mind the simulated drift height was increased to 4.2 m. At that snow depth, ablation was incomplete, and the snow lasted throughout the summer increasing in depth by about 10 cm/yr through the third year of simulated snow fence modification. It would therefore seem probable that a fence array producing drifting to a depth of over 4.2 m would be capable of initiating a permanent snowfield at Barrow. A permanent snowfield would offer interesting management possibilities as a freshwater source, since the radiative characteristics of the surface might be altered to match demand.
TABLE 3. Sensitivity Test, Urbanization

<table>
<thead>
<tr>
<th>Snow Albedo</th>
<th>Meltout Date</th>
<th>Snow Roughness Length, cm</th>
<th>Meltout Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.82</td>
<td>June 14</td>
<td>0.1</td>
<td>June 14</td>
</tr>
<tr>
<td>0.70</td>
<td>June 7</td>
<td>10.</td>
<td>June 11</td>
</tr>
<tr>
<td>0.60</td>
<td>June 6</td>
<td>50.</td>
<td>June 8</td>
</tr>
<tr>
<td>0.50</td>
<td>June 2</td>
<td>100.</td>
<td>June 7</td>
</tr>
</tbody>
</table>

TSKY radiant temperature of clear sky.
ZO roughness length of surface.
UA wind velocity.
AIRDEN air density.
EXCO atmospheric turbulent exchange coefficient.
BB( ) blackbody function.
RSKY thermal radiation flux from sky hemisphere.

URBANIZATION AND MELTOUT

As mentioned in the introduction, meltout is known to occur 2 weeks early in Barrow Village. Two mechanisms for this effect are proposed: lowered snow albedo (possible range 0.82-0.20) and increased aerodynamic roughness in the village (possible range 0.1-100 cm). These effects were sensitivity tested by using the 1971 weather, and the results are abstracted in Table 3.

The simulated system is more sensitive to reduced snow albedo than to increased aerodynamic roughness length. Thus of the two mechanisms, reduced albedo in the village due to dust fall, snow removal, vertical walls, etc., would appear to be a predominant mechanism in initiating early melt. This hypothesis needs the additional support of albedo measurements by aircraft and a detailed analysis of the aerodynamic roughness geometry of the village. At this stage the modeling simulation effort is employed as a design guide for critical experimentation.

CONCLUSION

This work has demonstrated the feasibility of constructing a generalized annual snow-soil temperature model of tundra and permafrost terrains. Further, the coupled equilibrium temperature-soil thermal diffusion model appears sufficiently generalized to act as the basis for specific annual simulations. Simulation experiments indicate that snow fence arrays producing drifts greater than 4.2 m in depth should produce late-lying snowbanks at Barrow and other locations in the Alaskan coastal plain. The simulator indicates that snow albedo reduction in Barrow Village is dominant over aerodynamic modification in initiating the melt effect.

NOTATION

Variable names in equations (2)-(4) are the same as those used in the Fortran 4 source code. Note also the use of Fortran algebraic and logical operators in equations (2)-(4).

Equation (1)

\[ T \] temperature.
\[ t \] time.
\[ l \] space node index.
\[ t^* \] reverse time.
\[ Z \] depth.
\[ D \] diffusivity.
\[ F \] Fourier modulus.

Equation (2)

\[ ZOS \] snow aerodynamic roughness length, 0.1 cm.
\[ ZOT \] tundra aerodynamic roughness length, 2 cm.
\[ XKAR2 \] Von Karman constant squared (0.16).
\[ ZA \] atmospheric computation level, 20 m.
\[ TAK \] air temperature, degrees Kelvin.
\[ Q \] specific humidity of air.

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