ABSTRACT

The implication of certain forms of depreciation on machinery repair functions are explored. Inconsistencies between farm machinery repair functions in the Agricultural Engineers Yearbook and commonly accepted depreciation schedules are presented.

Accurate machinery repair cost estimates are a prerequisite for a variety of decisions common in farm management. Machinery replacement, new versus used machinery purchase decisions, lease versus own decisions, and cost of production estimates are dependent upon repair cost.

The objective of this paper is to discuss conditions imposed upon the mathematical form of machinery repair function by generally accepted depreciation schedules. Repair functions presented in the Agricultural Engineers Yearbook [1] will be investigated to determine if the conditions are violated.

Castle et al., [2] state, "The amount of depreciation charged should correspond to the loss in value of the asset over time." Furthermore, Castle et al., go on to indicate that three commonly accepted depreciation methods are straight line (SL), sum of years digits (SD) and declining balance (DB). These depreciation methods were also legally acceptable for income tax purposes until 1981 when depreciation will be computed using a capital recovery factor which is based upon a combination of the declining balance and the straight line method. In mathematical form, these depreciation methods imply

\[ D(t) > 0 \quad \text{Condition 1} \]
\[ \frac{\partial D(t)}{\partial t} \leq 0 \quad \text{Condition 2} \]
\[ \frac{\partial^2 D(t)}{\partial t^2} \geq 0 \quad \text{Condition 3} \]

where \( D(t) \) is the depreciation or loss in machine value at machine age \( t \). Condition 1 implies that depreciation will be positive or that the machine loses value with age. Presumably the machine will be used and total accumulated usage will increase with age so Condition 1 could be restated in terms of usage. Eventually usage is expected to "wear out" a machine, and so the more a machine is used, the less remaining use is embodied in the machine resulting in declining value and positive depreciation. Of course unusual maintenance strategies or unexpected changes in prices of commodity produced by machinery may invalidate Condition 1. Condition 2 simply states that depreciation is either constant (straight line) or declining (i.e., declining balance or sum of years digits) and that if depreciation is declining it is declining at a decreasing rate (i.e., declining balance) or constant rate (i.e., sum of years digits). The Agricultural Engineers Yearbook [1] specifies remaining value (used price) functions as a portion of list (new) price in the following mathematical form

\[ C_1 C_2^t \]

where, \( C_1 \) and \( C_2 \) are constants between zero and one; and \( t \) is the machine age. The change in machine value (as a portion of list price) is

\[ \frac{\partial C_1 C_2^t}{\partial t} = C_1 C_2^t \ln C_2 = -D(t) \]

Since \( \ln C_2 < 0 \), then \( D(t) > 0 \) which is consistent with Condition 1. Conditions 2 and 3 are also satisfied by this function since

\[ \frac{\partial D(t)}{\partial t} = -C_1 C_2^t \ln C_2 < 0 \]
\[ \frac{\partial^2 D(t)}{\partial t^2} = -C_1 C_2^t \ln^2 C_2 > 0. \]

The value of any asset (machine) is equal to the asset’s imputed value\(^2\) or the discounted present value of the income the asset generates plus the asset’s discounted salvage value. Therefore

\(^2\)See Samuelson [5].
V(t) = \int Y \cdot R(i) \, e^{rt} \, di + V(n) \, e^{rn}

where:

V(i) = value of machine at age i
Y = the net returns generated by the machine excluding repairs, depreciation and opportunity cost on the machine
R(i) = annual repair costs of a machine at age i
r = discount rate (r > 0)
t = machine purchase age
n = machine selling age
i = machine age or time.

Equation 1 implies that the only variable which changes with machine age is repair cost since Y is independent of machine age and makes no allowance for uncertainty or obsolescence. Such an assumption is quite common, particularly in machinery replacement literature.3

Implications regarding the mathematical form of the machinery repair function can be developed from conditions 1, 2, and 3 and the imputed value function (Equation 1). First, it seems logical that annual repair costs are positive or R(t) > 0. R(t) > 0 will be referred to as Condition 1A.

Depreciation is equal to the change in value of the machine over time or

\[
\frac{\partial V(t)}{\partial t} = -Y + R(t) + r \, V(t)
\]

Since \(\frac{\partial V(t)}{\partial t}\) is generally less than zero (machine is declining in value) and depreciation [D(t)] is indicated as a positive number, D(t) will be defined as

\[
D(t) = -\frac{\partial V(t)}{\partial t} = Y - R(t) - rV(t).
\]

Equation 3 says that depreciation equals income less repair costs, less opportunity cost or cost equal benefits at any machine age.

From Equation 3

\[
\frac{\partial D(t)}{\partial t} = -\frac{\partial R(t)}{\partial t} - r \, \frac{\partial V(t)}{\partial t} = -\frac{\partial R(t)}{\partial t} + r \, \frac{\partial D(t)}{\partial t}
\]

\[
\frac{\partial R(t)}{\partial t} = r \, \frac{\partial D(t)}{\partial t} - \frac{\partial D(t)}{\partial t}
\]

which with Conditions 1 and 2 imply \(\frac{\partial R(t)}{\partial t} \geq 0\), which will be referred to as Condition 2A. From Equation 4

\[
\frac{\partial^2 R(t)}{\partial t^2} \leq 0,
\]

which will be referred to as Condition 3A. If Equation 1 and the three previously mentioned forms of depreciation hold, then repair functions must meet Conditions 1A, 2A, and 3A.

Repair functions from the Agricultural Engineers Yearbook [1] were analyzed to determine whether Conditions 1A, 2A and 3A were met. Repair functions from the Agricultural Engineers Yearbook are based upon accumulated usage, either in terms of total hours or acres. It seems reasonable that annual usage will be approximately constant during the first years of use. Older machines are often used as a "back-up" for newer machines and so are used less. The implications of age usage relationship will be discussed more, later in the paper. At this point, annual use will be assumed constant which implies that mathematical properties related to age and use are congruent.

The general form of the engineers repair function is

F(X) = \alpha X^\beta

where

F(X) = accumulated repair costs as a percent of purchase price
X = total usage divided by 1000
\alpha and \beta = constants.

Then

F(Ut) = \alpha(Ut)^\beta

where:

U = annual usage divided by 1000
t = age.

Annual repair costs are equal to \(\frac{\partial F(Ut)}{\partial t}\), defined as

R(U,t), or R(U,t) = \alpha \beta U t^{\beta-1}, Conditions 1A, 2A, and 3A are met if and only 1 \leq \beta \leq 2.4

\[
\frac{\partial R(U,t)}{\partial t} = \alpha (\beta - 1) U t^{\beta-2}
\]

which with Conditions 2 and 3 imply \(\frac{\partial^2 R(U,t)}{\partial t^2} \leq 0, \)

which will be referred to as Condition 3A. If Equation 1 and the three previously mentioned forms of depreciation hold, then repair functions must meet Conditions 1A, 2A, and 3A.

\[
\frac{\partial^2 D(t)}{\partial t^2} \leq 0,
\]

which will be referred to as Condition 3A. If Equation 1 and the three previously mentioned forms of depreciation hold, then repair functions must meet Conditions 1A, 2A, and 3A.

\[
\frac{\partial^2 R(U,t)}{\partial t^2} = (\beta - 1) (\beta - 2) U t^{\beta-3}
\]

If 1 \leq \beta \leq 2 then Conditions 1A, 2A, and 3A are met. If 2 < \beta < 3 then Condition 3A is violated. If 0 < \beta < 1 then Conditions 2A and 3A are violated.

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3See Perrin [4] and Chisholm [3].
Table 1 presents repair functions from the Agricultural Engineers Yearbook and indicates whether they meet the conditions previously presented. Many of the newer Agricultural Yearbook repair functions and all of the newer functions for self-propelled machinery (tractors and combines) do not meet Conditions 1A, 2A, or 3A. All of the earlier repair functions meet the conditions.

Inflation does not affect the analysis if all variables inflate at the same rate and the nominal discount rate is greater than the inflation rate. Equation 1 under inflation becomes

\[ V(t) = \int_1^n [Y-R(i)] e^{-r(t-i)} e^{r(t-n)} \, di + V(n) e^{-r(t-n)} \]

where:

- \( V(i) \) = value of machine at age \( i \) in time or machine age zero dollars
- \( Y \) = the net returns generated by the machine excluding repairs, depreciation and opportunity cost on the machine in time zero dollars
- \( R(i) \) = annual repair costs of a machine of age \( i \) in time zero dollars
- \( t \) = machine purchase age
- \( n \) = machine selling age
- \( r \) = nominal discount rate
- \( f \) = rate of inflation
- \( i \) = machine age or time.

Depreciation is equal to the negative of the change in value with age or

\[ \frac{\partial V(t)}{\partial t} = -Y + R(t) + (r-f)V(t) \]

which is identical to Equation 2 except that \( r-f \) has replaced \( r \). Therefore all results derived previously hold under inflation (assuming \( r-f > 0 \)).

A variety of reasons could account for the discrepancy between agricultural engineers repair functions and three conditions implied by commonly accepted depreciation schedules. The imputed value function (Equation 1) could be misspecified. Variables other than repair cost could be a function of machine age such as fuel consumption, property taxes, insurance, and crop loss caused by machine "down time." Property taxes and insurance are gen-

<table>
<thead>
<tr>
<th>Machine</th>
<th>Repair Function</th>
<th>Meet Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970 Midwest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tractors gasoline</td>
<td>0.0183 X(^2.159)</td>
<td>No</td>
</tr>
<tr>
<td>diesel</td>
<td>0.0120 X(^2.033)</td>
<td>No</td>
</tr>
<tr>
<td>LPG</td>
<td>0.0131 X(^2.122)</td>
<td>No</td>
</tr>
<tr>
<td>SP combine</td>
<td>0.052 X(^2.122)</td>
<td>No</td>
</tr>
<tr>
<td>Corn pickers</td>
<td>0.686 X(^2.346)</td>
<td>No</td>
</tr>
<tr>
<td>Moldboard plows</td>
<td>0.360 X(^1.910)</td>
<td>Yes</td>
</tr>
<tr>
<td>Disk harrows</td>
<td>0.012 X(^1.714)</td>
<td>Yes</td>
</tr>
<tr>
<td>Chisel plows and field cultivators</td>
<td>0.037 X(^1.420)</td>
<td>Yes</td>
</tr>
<tr>
<td>Row planters</td>
<td>0.093 X(^2.137)</td>
<td>No</td>
</tr>
<tr>
<td>Grain drills</td>
<td>0.089 X(^2.626)</td>
<td>No</td>
</tr>
<tr>
<td>Row cultivators</td>
<td>0.023 X(^2.207)</td>
<td>No</td>
</tr>
<tr>
<td>Rotary hoes</td>
<td>0.012 X(^1.396)</td>
<td>Yes</td>
</tr>
<tr>
<td>PTO windrower-conditioners</td>
<td>0.063 X(^1.595)</td>
<td>Yes</td>
</tr>
<tr>
<td>PTO forage harvesters</td>
<td>0.158 X(^0.185)</td>
<td>No</td>
</tr>
<tr>
<td>Stalk choppers</td>
<td>2.120 X(^0.904)</td>
<td>No</td>
</tr>
<tr>
<td>Earlier Nationwide</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crawler, 4-wheel drive tractors</td>
<td>0.0024(X)(^1.5)</td>
<td>Yes</td>
</tr>
<tr>
<td>Power units, 2-wheel drive tractors</td>
<td>0.0029(X)(^1.5)</td>
<td>Yes</td>
</tr>
<tr>
<td>SP combines, cotton pickers, forage harvesters, pull-type cotton strippers, rotary stalk cutters and pickup trucks</td>
<td>0.0364(X)(^1.5)</td>
<td>Yes</td>
</tr>
<tr>
<td>Floats and scrapers, land plane front end loaders, balers with engines, manure spreaders, feed trucks</td>
<td>0.0266(X)(^1.5)</td>
<td>Yes</td>
</tr>
<tr>
<td>Mounted cotton pickers and corn pickers, forage harvesters and blowers, flail harvesters, farm trucks and SP sprayers</td>
<td>0.0481(X)(^1.4)</td>
<td>Yes</td>
</tr>
<tr>
<td>PTO balers, sugarbeet and potato harvesters</td>
<td>0.0352(X)(^1.4)</td>
<td>Yes</td>
</tr>
<tr>
<td>PTO combines and corn heads for SP combines</td>
<td>0.0602(X)(^1.4)</td>
<td>Yes</td>
</tr>
<tr>
<td>SP swathers, rakes, hay conditioners, feed wagons</td>
<td>0.0441(X)(^1.4)</td>
<td>Yes</td>
</tr>
<tr>
<td>Seeding equipment, mounted sprayers</td>
<td>0.1232(X)(^1.4)</td>
<td>Yes</td>
</tr>
<tr>
<td>Wagons with boxes</td>
<td>0.0167(X)(^1.4)</td>
<td>Yes</td>
</tr>
<tr>
<td>Fertilizer distributing equipment</td>
<td>0.1222(X)(^1.3)</td>
<td>Yes</td>
</tr>
<tr>
<td>Cutterbar mowers and all tillage equipment</td>
<td>0.0915(X)(^1.3)</td>
<td>Yes</td>
</tr>
</tbody>
</table>

\(^1\)Schoney and Finner [6] indicate that under inflation machines depreciate less in the first years and more in later years in real terms. However, this may be attributed to the short data set available for high inflation years.
eral a linear function of machine value and so would not affect the implications of the analysis. If use is declining (not a linear function of age), the conditions may not be valid.\footnote{If all variables are defined as in Equation 1 and property tax and insurance are a percent of machine value, \( \delta \) then \( V(t) = \int_0^t [Y \cdot R(i) \cdot \alpha V(i)] e^{\alpha(i \cdot t)} di + V(m) e^{\alpha(m \cdot t)} \) \( \frac{\partial V(t)}{\partial t} = -Y + R(t) + (\alpha + \gamma) V(t) \). Therefore it is obvious that the previously discussed implications regarding repair functions will not be affected by costs which are a linear function of machine value.}

**SUMMARY**

If depreciation forms included in this analysis are not representative of market depreciation then the conditions developed for repair functions are not valid. For instance if Condition 3 is relaxed such that depreciation can be decreasing at an increasing absolute rate \( \frac{\partial D(t)}{\partial t^2} > 0 \), then only the PTO harvesters and stalk choppers would not meet the conditions. However, intuitively it seems that Condition 3A should hold or that depreciation should decrease at a decreasing absolute rate.

The discrepancy could be due to misspecification of the repair functions. If misspecification is the problem then more statistical analysis of the data may be required, or in light of the age of the data (1970), quite possibly new data should be gathered and analyzed. Such a project is particularly important due to the widespread use of these functions in farm management research.

\[ \frac{\partial^2 g(t)}{\partial t^2} < 0 \text{ or annual use declining} \]

Then
\[ \frac{\partial F(X)}{\partial t} \frac{\partial X}{\partial t} = \text{annual cost} \]
\[ \frac{\partial^2 F(X)}{\partial X^2} \frac{\partial X}{\partial t} \frac{\partial^2 F(X)}{\partial X^2} \frac{\partial X}{\partial t} = \text{change in annual cost} \]

The sign of \( \frac{\partial^2 F(X)}{\partial X^2} \) is indeterminate even if \( \frac{\partial F(X)}{\partial X} > 0 \) (Condition 2A) holds since
\[ \frac{\partial^2 X}{\partial t^2} = \frac{\partial^2 g(t)}{\partial t^2} < 0 \]
\[ \frac{\partial^2 X}{\partial t^2} = \frac{\partial^2 X}{\partial t^2} \]

**REFERENCES**


