Integral Method for Estimating Soil Hydraulic Properties

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ABSTRACT

Soil hydraulic properties are required to fully understand and predict soil water distribution. Soil hydraulic properties include the soil water characteristic curve and hydraulic conductivity. We used an integral method to solve the problem of water absorption into a horizontal soil column. The integral solutions to the problem were used to estimate the parameters \( \alpha \) and \( n \) in the van Genuchten model for soil water characteristic curves. The two parameters, \( \alpha \) and \( n \), in the characteristic curve model were estimated by the length of the wetted zone, sorptivity, and saturated hydraulic conductivity. This new integral method uses both Richards' equation and the closed-form equations of soil hydraulic properties. Six soils ranging from sandy loam to clay loam were used to test the method. Soil water characteristic curves estimated by the infiltration method are in good agreement with measured characteristic curves. The integral method provides a transient water flow approach to estimate the soil water characteristic curve instead of the usual equilibrium method. This is a new and simple means to determine soil hydraulic properties.

Increasing evidence shows that the quality of soil and water resources on the Earth is being adversely affected by the release of a variety of agricultural and industrial pollutants into the environment (van Genuchten, 1992). Water is the most important carrier of the pollutants into our soils. Rates of soil water movement in various soil water flow processes (e.g., infiltration, redistribution, root uptake, and drainage) are important for making practical soil management decisions to minimize potential groundwater contamination and degradation of soil quality from land-applied chemicals. Numerical solutions of the flow and transport problems in the vadose zone are the most important approaches to predict quantitatively the dynamic behavior of the system. Unsaturated flow and transport modeling usually requires accurate and complete information about the unsaturated hydraulic properties for the model to function properly. Soil hydraulic properties include a soil water characteristic curve (the relation between volumetric water content [\( \theta \)] and pressure head [\( h \)] \( \theta(h) \)), hydraulic conductivity \( (K) \), and water diffusivity \( (D) \). Because the three hydraulic properties are related by \( K = D \frac{dh}{dh} \), only two of them are independent. Usually the hydraulic conductivity and the soil water characteristic curve are considered to be two of the most important hydraulic properties.

The methods of determining unsaturated hydraulic properties are conveniently divided into two groups, direct methods and indirect methods (Neuman, 1973; van Genuchten, 1992). For the direct group, most methods for measuring soil hydraulic properties, i.e., the soil water characteristic curve and hydraulic conductivity, both in the laboratory and in situ, have been described by Green et al. (1986) and by Klute and Dirksen (1986). Although direct methods are relatively clear in concept, they have some limitations that restrict their use in practice (van Genuchten, 1992). Time consumption and uncertainty in the estimated hydraulic parameters are the common limitations for most direct methods, especially for field methods.

Because the direct determination of hydraulic properties is relatively time consuming and expensive, various efforts have been made to relate hydraulic conductivity and the soil water characteristic curve to easily determined soil physical properties. This approach results in indirect methods. For example, soil texture data were successfully used (Bouma and van Lanen, 1987; Puckett et al., 1985; Dane and Puckett, 1992; Tyler and Wheatcraft, 1989, 1990; Wosten et al., 1995) to predict soil water characteristic curves, which could subsequently be used to estimate hydraulic conductivity based on the models of Brooks and Corey (1964), Mualem (1976), and van Genuchten (1980). Recent application of indirect methods (Kool et al., 1987; Kool and Parker, 1988; Russo et al., 1991; Sisson and van Genuchten, 1991; Arya and Dierolf, 1992; Wu and Vomocil, 1992) have shown several advantages compared with the direct techniques (van Genuchten, 1992; van Dam et al., 1992).

Complete hydraulic property estimation across a wide range of soil water content and information on parameter uncertainty are major advantages of indirect methods. However, a number of problems, such as convergence and parameter uniqueness, related to indirect methods still remains to be solved (van Genuchten, 1992).

To remove some limitations from both direct methods and indirect methods, we developed an integral method for estimating soil hydraulic properties. The integral method is theoretically based on the Richard equation of water flow in soils and is practical, easy, and convenient in terms of the required measurements. The integral method gives approximate solutions to nonlinear partial differential equations (PDE). The essential idea of the integral method is to approximate the solution to the PDE with a simple function that contains adjustable parameters, and then determine the values of these parameters by requiring the solution to satisfy both the PDE and initial and boundary conditions in an integral sense. The integral method was first used to solve diffusion problems by Landahl (1953). There have been applications of this method in flow problems of porous media (Prasad and Romkens, 1982; Zimmerman and Bodvarsson, 1989; Zimmerman et al., 1990). We used the integral approach to solve the highly nonlinear hori-

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Abbreviations: ODE, ordinary differential equation; PDE, partial differential equations; RMSE, root mean square error.
horizontal water flow equation in soils and to estimate the parameter values of the van Genuchten (1980) soil hydraulic property models. We also compared the predicted and measured soil water characteristic curves for six soils ranging from sandy loam to clay loam.

**THEORY**

The equation describing one-dimensional horizontal unsaturated flow of water in soils is

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[ K(h) \frac{\partial h}{\partial x} \right]$$  \[1\]

where \(h\) is the volumetric soil water content (m³/m³), \(K(h)\) is the unsaturated hydraulic conductivity (m/s), \(x\) is the horizontal distance (m), and \(t\) is the time (s).

The initial and boundary conditions are similar to those of the Bruce and Klute (1956) water absorption problem. The only difference is that the variable to describe the initial and boundary conditions in Bruce and Klute (1956) water absorption is water content and the variable in current application is pressure head. Mathematically, these conditions are described as follows:

- \(h(x,0) = h_0\) \[2\]
- \(h(0,t) = 0\) \[3\]
- \(h(\infty, t) = h_i\) \[4\]

where \(h_i\) is the initial pressure head. Without loss of generality, a zero pressure head at the inlet boundary is assumed because the solution for a non-zero pressure boundary is related in a simple way (Philip, 1957) to the solution of zero inlet boundary. The Boltzmann transformation variable, \(\lambda = x/f\), is used to convert the PDE, Eq. [1], into an ordinary differential equation (ODE). With this transformation variable, Eq. [1] is transformed into

$$\frac{d}{d\lambda} \left[ K(h) \frac{dh}{d\lambda} \right] + \lambda \frac{d\theta}{2d\lambda} = 0$$  \[5\]

The initial and boundary conditions (Eq. [2]-[4]) are converted to

- \(h(0) = 0\) \[6\]
- \(h(\infty) = h_i\) \[7\]

By performing the transformation, the mixed problem of PDE (Eq. [1]-[4]) is reduced to a two-point ODE boundary value problem, given by Eq. [5] through [7]. In Eq. [5], there are two variables, \(h\) and \(\theta\); an additional equation (the soil water characteristic curve) that relates the two variables is needed to solve the two-point ODE boundary value problem.

The most commonly used closed-form equations for characterizing the soil water characteristic curve and hydraulic conductivity in soil physics are those of van Genuchten (1980) and Mualem (1976). The equations are

- \(\theta = \theta_r + (\theta_s - \theta_r)[1 + (\alpha |h|)^{n}]^{-m} \)  \[8\]
- \(K(h) = K_s [1 - (\alpha |h|)^{n-1} (1 + (\alpha |h|)^{n})^{-m}] \)  \[9\]

where \(\theta(h)\) is volumetric water content as a function of water pressure head, \(\theta_s\) is saturated water content, \(\theta_r\) is the residual water content, \(\alpha\) is a scaling parameter that is inversely proportional to the mean pore diameter, \(1/\alpha\) is similar to air-entry pressure in the model of Brooks and Corey (1964, 1966), and \(n\) is the soil water characteristic curve index (shape parameter of the curve) or the pore-size distribution index, \(K_s\) is the saturated hydraulic conductivity, and \(m = 1 - 1/n\).

An appropriate water content profile that can be described by a simple form of function, \(\theta(\lambda)\), may be obtained by the following reasoning. The flux of water infiltrating into the soil is finite. That means \(dh/d\lambda\) must be finite at \(\lambda = 0\). It is convenient to express the relationship, \(h(\lambda)\), in terms of MacLaurin's series, i.e.

$$h = a_0 + a_1\lambda + a_2\lambda^2 + \ldots$$  \[10\]

Because \(h(0) = 0\), this means \(a_0 = 0\), then \(h = a_1\lambda + a_2\lambda^2 + \ldots\). \(a_1\) is a negative constant. Again \(h\) is usually negative. For convenience, let \(b_1 = -a_1\); then \(b_1\) is a positive constant. Substitution of this into Eq. [8] to make the first-order approximation provides a simple representative form of the water distribution as

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = 1 - m(\alpha b_1 \lambda)^n$$  \[11\]

The length of wetted zone is denoted by \(d\). In order to find \(b_1\), we use the condition at \(\lambda = d\), \(\theta(d) = \theta_s\), so the term \(mb_1^n\) is given by

$$mb_1^n = \frac{\theta_s - \theta_r}{(\theta_s - \theta_r)(\alpha d)^n}$$  \[12\]

Combining Eq. [11] and [12] gives the appropriate water content profile (also see Fig. 1):

$$\theta(\lambda) = \theta_s - (\theta_s - \theta_r)(\alpha d)^n \quad 0 < \lambda < d$$  \[13\]

$$\theta(\lambda) = \theta_s \quad d \leq \lambda < \infty$$  \[14\]

Equation [13] describes an absorption profile the same as the exact solution described by Philip (Philip, 1960, Table 1, no. 2):

$$\lambda(\Theta) = \varepsilon(1 - \Theta)p \quad p > 0$$  \[15\]

where \(\Theta\) is normalized volumetric water content, \((\theta - \theta_r)/((\theta_s - \theta_r))\), \(\varepsilon\) is the maximum value of \(\lambda\), the same as \(d\) in Eq. [13], and \(p\) is the slope factor. If \(p = 1/n\), one can verify that Eq. [13] and [15] are equivalent. This substantiates the water content profile described by Eq. [13] and [14]. Equation [13] will also be verified by experimental evidence.

The characteristic wetting length, \(d\) (referred to here simply as wetting length), can be related to the parameters of van...
Table 1. Some physical properties of the five soils.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Specific surface (10^{-3} \text{ m}^3/\text{kg})</th>
<th>Particle density (\text{Mg/m}^3)</th>
<th>Saturated water content (\text{m}^3/\text{m}^2)</th>
<th>Residual water content (\text{m}^3/\text{m}^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy loam</td>
<td>-</td>
<td>2.67</td>
<td>0.440</td>
<td>0.000</td>
</tr>
<tr>
<td>Silt loam</td>
<td>41</td>
<td>2.69</td>
<td>0.562</td>
<td>0.118</td>
</tr>
<tr>
<td>Loam</td>
<td>40</td>
<td>2.69</td>
<td>0.503</td>
<td>0.130</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>59</td>
<td>2.64</td>
<td>0.542</td>
<td>0.115</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>79</td>
<td>2.67</td>
<td>0.562</td>
<td>0.163</td>
</tr>
<tr>
<td>Clay loam</td>
<td>141</td>
<td>2.57</td>
<td>0.569</td>
<td>0.182</td>
</tr>
</tbody>
</table>

Genuchten’s (1980) model by integrating Eq. [5] from \(\lambda = 0\) to \(\lambda = \infty\) with Eq. [13] and [14] substituted for \(\theta(\lambda)\). The first term in Eq. [5] is

\[
\int_{0}^{\infty} \frac{d}{d\lambda} \left[ K(h) \frac{dh}{d\lambda} \right] d\lambda = \left[ K(h) \frac{dh}{d\lambda} \right]_{0}^{\infty} = b_{i}K_{i} \tag{16}
\]

which is derived based on the fact that at \(\lambda = \infty\), \(d\lambda/d\lambda = 0\), and at \(\lambda = 0\), \(d\lambda/d\lambda = -b_{i}\). From Eq. [12], \(b_{i}\) is expressed as

\[
b_{i} = \frac{1}{2\alpha} \left[ \frac{\theta_{i} - \theta_{i,0}}{m(\theta_{i} - \theta_{i,0})} \right]^{\alpha} \tag{17}
\]

The second term in Eq. [5] is

\[
\int_{0}^{\lambda} \frac{\lambda}{2} \frac{d\theta}{d\lambda} d\lambda = \int_{0}^{\lambda} \frac{\lambda}{2} \frac{d\theta}{d\lambda} d\lambda = \frac{-n(\theta_{i} - \theta_{i,0})d}{2(n + 1)} \tag{18}
\]

Equation [18] is obtained by the fact that the integral is zero in the interval \((d, \infty)\) since \(d\lambda/d\lambda \) is zero in this interval.

Combining Eq. [16], [17], and [18] with the integral of Eq. [5] gives

\[
\alpha = \frac{2(n + 1)K_{i}}{n(\theta_{i} - \theta_{i,0})d^{2}} \left[ \frac{\theta_{i} - \theta_{i,0}}{m(\theta_{i} - \theta_{i,0})} \right]^{\alpha} \tag{19}
\]

From Eq. [19], the scaling parameter, \(\alpha\), is related not only to \(d\) but also to \(K_{i}\) and \(n\). In order to estimate both \(\alpha\) and \(n\), one more relation is needed if \(K_{i}\) is available (usually \(K_{i}\) is measured). This is obtained by applying Darcy’s flux equation to the horizontal absorption. At \(x = 0\) (the inlet boundary), the water flux is expressed as

\[
q = -\left[ K(h) \frac{dh}{dx} \right]_{x=0} \tag{20}
\]

and

\[
\frac{dh}{dx} \bigg|_{x=0} = \left( \frac{dh}{dx} \right)_{x=0} \left( \frac{dh}{dx} \right)_{x=0} \bigg|_{x=0} \tag{21}
\]

From Eq. [10], one can get

\[
\frac{dh}{dx} \bigg|_{x=0} = a_{i} = -b_{i} \tag{22}
\]

Considering \(K(h) = K_{i}\) at \(x = 0\) and the definition of the Boltzmann variable, and combining Eq. [20], [21], and [22] gives

\[
q = K_{i}b_{i}t^{0.5} \tag{23}
\]

For horizontal infiltration, if one uses Philip’s two-term equation, the infiltration rate or flux density is given by

\[
q = \frac{S}{2t^{0.5}} \tag{24}
\]

where \(S\) is sorptivity that can be relatively easily obtained by analyzing the infiltration rate with time (a simple regression will find \(S\) by using Eq. [24]). Combining Eq. [17], [23], and [24] gives

\[
\alpha = \frac{2K_{i} \left[ \frac{1( \theta_{i} - \theta_{i,0})}{m(\theta_{i} - \theta_{i,0})} \right]^{\alpha}}{Sd} \tag{25}
\]


\[
n = \frac{S}{d(\theta_{i} - \theta_{i,0}) - S} \tag{26}
\]

Equations [19] and [26] complete the parameter estimation for the van Genuchten (1980) model of hydraulic properties of a soil. First, \(n\) is obtained by measuring the characteristic wetting length and sorptivity. Then, with a \(K_{i}\) measurement, the scaling parameter, \(\alpha\), is found by using Eq. [19]. Experimentally, if one records both infiltration and wetting front with time in a horizontal absorption experiment and measures the saturated hydraulic conductivity of the column after absorption, the parameter estimation for the van Genuchten (1980) model can be completed because saturated and residual water content are easy to measure or estimate.

MATERIALS AND METHODS

Six soils were used to test the integral approach in this study. The first five soils are a silt loam obtained from land mapped as Flaggler series (a coarse-loamy, mixed, mesic Typic Hapludoll, 11.4% sand, 70.0% silt, and 18.6% clay), Nicollet loam (a fine-loamy, mixed, superactive, mesic Aquic Hapludoll, 50.9% sand, 32.6% silt, and 16.5% clay), Keswick sandy clay loam (a fine, smectitic, mesic Aquertic Chromic Haplu-doll, 67.7% sand, 11.3% silt, and 21.0% clay), Monona silty clay loam (a fine-silty, mixed, superactive, mesic Typic Endoaquoll, 24% sand, 69.5% silt, and 28.1% clay), Webster clay loam (a fine-loamy, mixed, superactive, mesic Typic Endoaquoll, 32.1% sand, 39.2% silt, and 28.7% clay). The sixth soil is a Manawatu fine sandy loam (a Dystric Fluvic Eutrochrept).

Information on the soil water characteristic curve of the sixth soil was obtained from Clothier and Scottor (1982).

Some basic physical properties of the first five soils were measured. The specific surface areas were measured by using the ethylene glycol monooethyl ether (EGME) technique (Cihacek and Brenner, 1979; Carter et al., 1986). Particle densities were determined by using the pycnometer method (Blake and Hartge, 1986b). Bulk densities were determined by the clod method (Blake and Hartge, 1986a). The saturated water contents of the first five soils were obtained by measuring both mass water contents and their bulk densities at saturation. The residual water contents were estimated as the water contents at -1.5 MPa matric pressure (van Genuchten, 1980).

The soil water characteristic curves for the first five soils were measured by pressure plate technique. Additionally, conventional horizontal-infiltration experiments of the Bruce and Klute (1956) type were performed for the determination of the characteristic depth \(d\) and sorptivity \(S\). Air-dried soil was packed into sectioned Plexiglas tubes 0.15 m long (15 sections) and 0.038 m in diameter with a controlled bulk density of 1.30 Mg/m³. During infiltration, water was supplied to one end of the soil column through a ceramic plate. During the horizontal infiltration (absorption), the advance of the wetting front with time and the amount of water infiltrated into the soil column were recorded. The horizontal absorption experiment was ended when the wetting front reached about half length of the column. The saturated hydraulic conductivities of the first five soils were measured by a constant-head technique (Klute and Dirksen, 1986).
DISCUSSION

Equations [19] and [26], representing the parameter estimation of the van Genuchten (1980) model of soil hydraulic properties, depend on six parameters, $K_s$, $S$, $d$, $\theta_s$, $\theta_i$, and $\theta_r$. The two water contents, $\theta_s$ and $\theta_i$, are easy to measure as the infiltration boundary and initial condition. The value of $\theta_i$ needs to be estimated (for example taking the water content at $-1.5$ MPa matric pressure as $\theta_i$). The characteristic length ($d$) of the wetted zone is easy to observe visually during infiltration. The value of $S$ is also relatively easy to determine from infiltration data. The only parameter left to determine is $K_s$, which can be conveniently measured by using the same soil column after the absorption experiment.

Equation [13] is used to derive Eq. [19] and [26]. As mentioned above, Eq. [13] is approximate. Mathematically, Eq. [13] and [15] are the same. Equation [15] should cover most absorption profiles of soil water content (Clothier et al., 1983). However, testing of Eq. [13] by measured data of soil water content distribution profiles should be performed. To test Eq. [13], measured data for 10 soils ranging from sand to clay were taken from the literature. The 10 soils are: Hagener sand (Selim et al., 1970), Hayden sandy loam (Whisler et al., 1968), Manawatu fine sandy loam (Clothier et al., 1983),
Adelanto loam (Jackson, 1963), Edina silt loam (Selim et al., 1970), Nicollet sandy clay loam (McBride and Horton, 1985), Fayette silty clay loam (McBride and Horton, 1985), Panoche clay loam (Reichardt et al., 1972), Pine silty clay (Jackson, 1963), and Yolo clay (Noziger, 1978). Figure 2a to 2j present the fit of Eq. [13] to soil water distribution data for these 10 soils. From Fig. 2, one can see that Eq. [13] is appropriate for describing the soil water distribution of a horizontal absorption experiment. In the derivation of Eq. [19] and [26], Eq. [13] is the only approximate expression. Both theoretical and experimental verifications of Eq. [13] give confidence that Eq. [19] and [26] should be appropriate for parameter estimations of the van Genuchten (1980) model of a soil water characteristic curve.

Equation [19] should be examined carefully. The scaling parameter, $a$, is directly proportional to $K_s$ for a given $n$. Larger values of $K_s$ generally correspond to coarser textured soils, therefore to larger $a$ values. The air-entry pressure is inversely proportional to $a$. This implies that the coarser textured soils have smaller air-entry values than do finer textured soils. This conclusion is expected.

It is necessary to make a sensitivity analysis of $a$ to $\theta$, because $\theta$ is approximated by the volumetric water content of $-1.5$ MPa water pressure of the soil. Assume that $n = 3$, $S = 1.5 \text{ mm/s}^{0.5}$, $K_s = 0.01 \text{ mm/s}$, $d = 6 \text{ mm/s}^{0.5}$, $\theta_s = 0.44$, and $\theta_r = 0.02$ (the soil may correspond to sandy loam). The sensitivity of $a$ to $\theta$, is shown in Fig. 3. From Fig. 3 it can be concluded that the effect of $\theta_r$ on $a$ is small. For example, an increase in $\theta_r$ from 0.01 to 0.2 corresponds with an increase in $a$ from 2.52 to 3.07 m$^{-1}$. That implies that a change in $\theta_r$, of a factor of 20 only makes a 20% change in $a$. Therefore an approximation of $\theta_r$ should not produce a large error in the scaling parameter.

The shape parameter, $n$, in the van Genuchten (1980) model is related both to sorptivity ($S$) and the characteristic length ($d$) of the wetted zone. Clothier and Scotter (1982) provided experimental evidence that, for a given soil, the relationship $\theta(X)$ is unique regardless of the duration of the sorption experiment. This means that $d$ is a constant for a given soil. Therefore $d$ is an indicator of hydraulic properties of a soil, and different soils have different values of $d$ (also see McBride and Horton, 1985). For a given $d$ (here, $d = 6 \text{ mm/s}^{0.5}$, $\theta_s = 0.5$, and $\theta_r = 0.05$), $n$ is proportional to $S$ (also see Fig. 4). An overestimation of $S$ will lead to an overestimation of $n$. An error in estimating $n$ will also result in an error in estimating $a$. The accuracy of estimating both $a$ and $n$ depends mainly on the accuracy of sorptivity estimation for a given soil because the determination of $K_s$, $\theta_s$, $\theta_r$, and $\theta$ should not produce large errors. Sorptivity estimation by fitting Eq. [24] to observed infiltration values is straightforward and hopefully does not allow a large error either. Therefore this integral method for estimating soil hydraulic properties should be accurate. Below, comparisons between soil water characteristic curves estimated by integral method and measured soil water characteristic curves are made to provide experimental confidence for the method.

To give more information about the soils, the particle size, shape parameter, and residual ($\theta_r$) and saturated ($\theta_s$) water contents for the six soils are presented in Table 3.

<table>
<thead>
<tr>
<th>Soil</th>
<th>$a$ (m/s)</th>
<th>$n$</th>
<th>$\theta_s$ (mm)</th>
<th>$\theta_r$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy loam</td>
<td>2.65</td>
<td>3.15</td>
<td>0.44</td>
<td>0.00</td>
</tr>
<tr>
<td>Silt loam</td>
<td>4.97</td>
<td>1.45</td>
<td>0.50</td>
<td>0.12</td>
</tr>
<tr>
<td>Loam</td>
<td>5.13</td>
<td>1.53</td>
<td>0.65</td>
<td>0.00</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>3.51</td>
<td>1.59</td>
<td>0.54</td>
<td>0.12</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>0.51</td>
<td>1.79</td>
<td>0.56</td>
<td>0.16</td>
</tr>
<tr>
<td>Clay loam</td>
<td>1.12</td>
<td>1.71</td>
<td>0.56</td>
<td>0.18</td>
</tr>
</tbody>
</table>
Fig. 5. Comparison of water characteristic curves obtained by the integral method (dashed curve) and by curve-fitting (solid curve) with observed data (filled square) for (a) fine sandy loam, (b) silt loam, (c) loam, (d) sandy clay loam, (e) silty clay loam, and (f) clay loam.

densities ($\rho_s$) and specific surfaces (SS) of the first five soils, together with $\theta_s$ and $\theta_r$ of all six soils are listed in Table 1.

The $\theta_s$ for fine sandy loam was obtained by averaging the first three measured water contents near saturation from the data of Clothier and Scotter (1982, their Fig. 1) because the water content near saturation seemed to be irregular. The $\theta_s$ was assumed to be zero for the fine sandy loam. This assumption may be safe for such a coarse-textured soil, and by using the van Genuchten (1980) model, the regression results of the water characteristic curve also gave a zero $\theta_r$.

The three important parameters for estimating $\alpha$ and $n$ are $S$, $K_s$, and $d$ (values of the Boltzmann variable at wetting fronts). Here they are referred to as hydraulic parameters. The values of the hydraulic parameters for
the six soils are shown in Table 2. The parameters of the sandy loam were taken from the literature (Clothier and Scotter, 1982; Clothier and Wooding, 1983).

An example of calculating \( n \) and \( \alpha \) may be helpful. For sandy loam (also see Table 2), the value of \( S \) is found to be \( 14.7 \times 10^{-4} \text{ m/s} \). The \( S \) values are obtained by curve-fitting Eq. [24] to the observed infiltration data; the value of \( d \) is \( 6.05 \times 10^{-3} \text{ m/s}^{1/2} \) (the value of the Boltzmann variable obtained from the wetting front at the end of the infiltration experiment); the value of \( K_s \) is \( 112 \times 10^{-2} \text{ m/s} \) (the \( K_s \) values for the other soils are measured by the constant-head technique [Dirksen, 1986]); the values of \( \theta_e, \theta_r, \) and \( \theta_h \) are 0.44, 0.00, and 0.08, respectively. Using Eq. [26] and the above values of \( d, S, \theta_e, \) and \( \theta_h \), \( n \) is found to be 3.15; using Eq. [19] and the above values of \( K_s, d, \theta_e, \theta_h, \) and \( \theta_h \), \( \alpha \) is found to be 2.65 m^{-1}.

Table 2 shows that both \( S \) and \( d \) tend to decrease when soils become finer in texture. With measured values of the parameters \( K_s, S, d, \theta_e, \theta_h, \) and \( \theta_h \), parameters \( \alpha \) and \( n \) can be determined from Eq. [19] and [26].

The calculated values of \( \alpha \) and \( n \) from the integral method for all six soils, together with those determined by curve-fitting the actual water characteristic curve data with the van Genuchten (1980) model, are listed in Table 3. The measured \( \theta_e \) and estimated \( \theta_h \), along with those obtained by curve-fitting are also included in Table 3. The \( \alpha \) and \( n \) values from both the integral method and from curve-fitting (van Genuchten, 1980) show similar trends of decreasing from sandy loam (coarser texture) to clay loam (finer texture). In general, curve-fit values of \( \theta_e \) are consistently lower than the estimated values of \( \theta_h \), for each soil. The fitted residual water contents do not have a clear relationship to soil textures.

The soil water characteristic curves of the first five soils estimated from the integral method are compared with those measured by pressure plate technique (Fig. 5b–5f). Comparison data (Fig. 5a) for the sixth soil (sandy loam) are taken from the literature (Clothier and Scotter, 1982). The fitted characteristic curves for all six soils, obtained by fitting the closed-form equation of van Genuchten (1980) to the observed data, are also shown in Fig. 5a to 5f. Generally, the soil characteristic curves estimated by the integral method are in good agreement with the observed data for all six soils. The estimated characteristic curves for the first five soils tend to overestimate water contents in the range of 0 to \(-1 \text{ m} \) in pressure head and underestimate water contents for the range of \(-1 \text{ to } -10 \text{ m} \) in pressure head. When pressure head is less than \(-10 \text{ m} \), the estimated characteristic curves compare well with the measured ones. The estimated curves cross with the fitted curves somewhere between \(-10 \text{ and } -1000 \text{ m} \) pressure head.

A statistical comparison of the results was carried out to find out the accuracy of the integral method. An objective and quantitative measure, root mean square error (RMSE) (Willmott et al., 1985), was used to estimate the accuracy. The RMSE values of the integral method along with those of curve fitting by using the van Genuchten (1980) model are listed in Table 4. The results show that the mean value of RMSE for the integral method is 0.034 while that for the curve fitting is 0.019. This result is expected because the curve-fitting method uses the measured data directly.

### Table 4. Root mean square errors (RMSE) of the integral method and curve-fitting technique.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Integral method</th>
<th>Curve fitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy loam</td>
<td>0.018</td>
<td>0.017</td>
</tr>
<tr>
<td>Silt loam</td>
<td>0.031</td>
<td>0.018</td>
</tr>
<tr>
<td>Loam</td>
<td>0.033</td>
<td>0.021</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>0.052</td>
<td>0.020</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>0.030</td>
<td>0.020</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.029</td>
<td>0.019</td>
</tr>
<tr>
<td>Mean</td>
<td>0.034</td>
<td>0.019</td>
</tr>
</tbody>
</table>

### CONCLUSION

The integral method has been used to give closed-form approximate solutions to the problem of horizontal absorption. Solutions are used to estimate the parameters of hydraulic property models of Mualem (1976) and van Genuchten (1980). With a horizontal absorption experiment, \( K_s \) is the only parameter needed to give complete information on hydraulic properties of a soil. The curve index, \( n, \) is estimated by \( d \) and \( S \), which can be measured in the horizontal absorption procedure. The scaling parameter, \( \alpha \), is estimated by \( K_s, d, \) and \( S \). The approximate solutions presented here show in theory how to estimate soil water characteristic curves and unsaturated hydraulic conductivity from simple horizontal infiltration experiments.

The experimental evidence provided in this study also shows that the integral method can be used to estimate soil hydraulic properties. The soil water characteristic curves estimated by the integral method are in good agreement with those observed for all six soils. Several weeks are needed to measure the water characteristic curves of six soils by using pressure plate equipment, whereas estimates can be accomplished with the integral method in several days by using very simple equipment (a horizontal infiltration device). The infiltration method can simultaneously estimate both the soil characteristic curve and unsaturated hydraulic conductivity from a horizontal infiltration experiment. Therefore, the integral method does not need specialized and expensive equipment and does not require substantial special operation skills either. The new method provides an attractive approach for estimating soil hydraulic properties.

### REFERENCES


