Application of surface-subsurface flow coupled with numerical simulator to runoff analysis in an actual field

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Abstract:
A numerical simulator that treats three dimensional, multiphase, surface-subsurface combined flows was applied to actual runoff analysis. A sensitivity study of hydraulic surface/subsurface interaction parameters required in numerical simulation was carried out utilizing short-term data, and a set of parameters that reproduces an actual hydrograph was obtained. To improve reproducibility of the hydrograph, a modification of discretization in expressing riverbed was introduced, so that a more realistic hydrograph is reproduced and a better quantitative estimation about base and side flow is realized. Applying the result of the sensitivity study to long-term runoff analysis, the hydrograph in one year was reproduced satisfactorily with the same hydraulic parameters obtained in the short-term runoff analysis. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS runoff analysis; numerical simulation; surface-subsurface combined flow

INTRODUCTION
In order to predict hydrological influences of large scale artificial activities on hydrological systems, it is important to consider the interaction between surface flow and subsurface flow. In conventional numerical analyses of groundwater and surface runoff, these two types of flow have been treated by lumped or distributed models (Ambroise et al., 1996; Refsgaard and Knudsen, 1996). However, almost all existing methods are surface flow oriented, and the treatment of physical modelling of three-dimensional groundwater flow is insufficient.

To analyse surface and subsurface flows simultaneously, the authors have developed a multi-phase flow type coupling method (Tosaka et al., 1996). In this method, the surface flow equation that is based on the dynamic wave approximation of a momentum equation is transformed to a type of two-phase groundwater flow equation, and introduced to a three-dimensional air-water two-phase subsurface flow simulator.

For the application of the method to runoff analysis, topographic, geological, and hydraulic parameters must be identified to reproduce an actual hydrograph from actual precipitation. In this paper, details of 3-D numerical modelling in a catchment area are introduced, and several numerical experiments carried out for a sensitivity study of input parameters with actual measured data in the experimental catchment area. A modelling technique for riverbeds is also introduced to reproduce surface-subsurface fluid interaction more appropriately.

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A fully coupled surface-subsurface flow treatment has been reported by Tosaka et al. (1996), Tosaka (1998). The formulation is briefly introduced here.

Based upon an immiscible two-phase flow formula applied to the air-water flow, the flow rate of the water phase in both saturated and unsaturated zone is described as

$$Q = -\frac{Kk_{rw}}{\mu_w B_w} \frac{\partial \Psi_w}{\partial x}$$  \hspace{1cm} (1)$$

where, $K$ = intrinsic permeability of soil or rock [L$^2$]; $k_{rw}$ = relative permeability of water [-]; $\mu_w$ = viscosity of water [ML$^{-1}$T$^{-1}$]; $B_w$ = formation volume factor of water that denotes the compressibility of fluid; $\Psi_w$ = hydraulic potential of water [ML$^{-1}$T$^{-2}$].

The hydraulic potential of the water phase is expressed as

$$\Psi_w = P_g - P_{cw} - \rho_w g z$$  \hspace{1cm} (2)$$

where, $P_{cw}$ = capillary pressure between each phase [ML$^{-1}$T$^{-2}$]; $\rho_w$ = density of water [ML$^{-3}$]; $g$ = gravity acceleration [LT$^{-2}$]; and $Z$ = depth [L].

For the surface flow, Manning’s formula is used for the calculation of velocity as

$$|v| = \frac{n}{R} R^{2/3} |I_f|^{1/2}$$  \hspace{1cm} (3)$$

where, $n$ = Manning’s coefficient, $R$ = hydraulic radius; $v$ = velocity; $I_f$ = friction slope.

Here, assuming an open channel with a rectangular cross section of width $W$ and depth $h$, $R$ can be expressed as

$$R = \left(\frac{hW}{2h + W}\right)$$  \hspace{1cm} (4)$$

From the dynamic wave approximation, the gradient of friction can be expressed as

$$I_f \approx -\frac{\partial h}{\partial x} + x = -\frac{1}{\rho_w g} \left[\frac{\partial P_w}{\partial x} - \rho_w g \frac{\partial z}{\partial x}\right] = -\frac{1}{\rho_w g} \frac{\partial \Psi_w}{\partial x}$$  \hspace{1cm} (5)$$

Using the dynamic wave approximation, the flow rate can be calculated from the following equation.

$$Q = vhW = \frac{R^{2/3}}{n} hW \sqrt{|I_f|}$$

$$= \frac{1}{n} \left[\frac{hW}{2h + W}\right]^{2/3} hW \sqrt{|I_f|}$$

$$= -\frac{1}{\rho_w g n} \left[\frac{WH}{2H + W}\right]^{2/3} \left[\frac{2H + W}{2S_{sw} H + W}\right]^{2/3} WH \sqrt{\rho_w g \frac{\partial \Psi_w}{\partial x}}$$

$$= -\frac{K^* k_{rw}^*}{\mu_w B_w} A^* \sqrt{\rho_w g \frac{\partial \Psi_w}{\partial x}}$$  \hspace{1cm} (6)$$
Here,

\[ K^* = \frac{\mu_n B_w}{\rho_w g n} \left[ \frac{WH}{2H + W} \right]^{2/3} \]

\[ A^* = WH \]

Equation (6) can be further linearized into the following equation (see Tosaka et al., 1996).

\[ Q = v h W \approx \frac{1}{n} \frac{R^{2/3}}{|x|^{1/2}} h W I \]

\[ = \frac{1}{n|z|^{1/2}} h W I \left[ \frac{WH}{2H + W} \right]^{2/3} \]

\[ = -\frac{1}{\rho_w g n |z|^{1/2}} \left[ \frac{WH}{2H + W} \right]^{2/3} S_w^5 \left[ \frac{2H + W}{2S_w H + W} \right]^{2/3} WH \frac{\partial \psi_w}{\partial x} \]  

(7)

Here, the notations in Equation (7) are

\[ K^* = \frac{\mu_n B_w}{\rho_w g n |z|^{1/2}} \left[ \frac{WH}{2H + W} \right]^{2/3} \]

\[ A^* = WH \]

The basic form of Equation (7) is the same type as Equation (1) that is used in the subsurface flow simulation. From this approximation, we can carry out a combined analysis of the surface-subsurface flow. The applicability of these formulations were validated by using the data from laboratory scale runoff experiments and field scale modelling. The fully dynamic wave model can also be combined with a subsurface flow simulation with a nonlinear analysis. In our simulator, both the fully dynamic wave model and the linearized dynamic wave model can be used. In special cases, such as overflows from levees, the linearized model can not be applied. However, in general cases, the linearized model gives an almost identical performance to the fully dynamic wave model with a shorter CPU time. For the subsequent simulation study, we used the linearized model.

OUTLOOK OF SITE AND SIMULATION STUDY

The numerical simulator for the surface-subsurface flow modelling was applied to Kiryu Experimental Catchment of Kyoto University. In this section, we give a summary of the site and the numerical model settings.

Outlook of site

A geographical map of the test site is shown in Figure 1 with the catchment boundary and main flow path. The total catchment area of the studied site is 49,900 m², average elevation is 234 m, and average precipitation per year is 1738 mm. The highest elevation in this area is 258.1 m, and the flow rate is measured at the outlet of this area.

In this area, the Forest Hydrology Group of Kyoto University has measured precipitation, river flow rate, and tested some hydraulic properties of surface soil. In this study, two types of measured rainfall and flow rate data are used for numerical modelling. For the parametric study, five days rain and flow rate data in 1995,
which shows the maximum short-term rainfall during 1995, were used. The actual utilized data are shown in Figure 2. During the five days, relatively heavy rainfall occurred four times, and four peaks of flow rate were measured. The main characteristic of the hydrograph in this area is that a relatively quick reduction of flow rate is observed after rainfall, and, the base flow shows a stepwise increase after each rainfall. Figure 3 shows the daily rainfall and flow rate in 1995.
Three dimensional numerical model

In this numerical simulation, the surface and subsurface region of the studied area was discretized into a flexible Finite Difference Grid system as shown in Figure 4, in which the first layer of the grid system is the atmosphere layer that keeps atmospheric pressure, the second layer is the surface flow layer, the third layer is the surface soil layer, and the rest are the base rock layers. The total number of grid blocks becomes \(39 \times 39 \times 5 = 7605\). In this grid system, the ground surface and riverbed are modelled in the second layer (the second Layer River model, called the 2LR model hereafter) as illustrated in Figure 5.

For the boundary condition, three boundary lines of the model set along a ridge are assumed as a no-flow boundary for both the surface and subsurface flow, and another side boundary including a measurement point is set as the constant pressure boundary. The upper side of the model is set as constant pressure, and the lower boundary is a no-flow boundary. For setting the initial condition, the authors carried out a transient numerical calculation with constant precipitation until the flow rate became stable and comparable with the measured value.

Figure 3. Measured daily rainfall and flow rate in 1995

Figure 4. Discretized Finite Difference grid system
Parameter setting

For hydraulic parameters, soil thickness, Manning’s coefficient, relative permeability, capillary pressure, and evapotranspiration rate are effective parameters in our runoff analysis.

Soil thickness is one of the most effective parameters to control a hydrograph, but actual distribution of soil thickness was not measured. In this model, the thickness of the soil layer (the third layer) was fixed at 3 m as the average thickness measured at several points in this area (Kubota et al., 1983; Kim et al., 1988).

Several other hydraulic parameters were also fixed according to actual field measurements (Kubota et al., 1983, 1987; Kim et al., 1988; Ohte et al., 1989). Their values used in this study are shown in Table I.

For Manning’s coefficient, the authors fixed this at 0.04 for the river channel, and set several values on the surface except for the river channel as a sensitivity check.

Table I shows the relative permeability between surface and soil which is a function of saturation of the surface grid blocks. For subsurface flow, many kinds of relative permeability models have been studied (Brooks and Corey, 1966; van Genuchten, 1980). However, numerical approaches for treating the interaction between surface and subsurface region have not been reported. This parameter should be determined from laboratory or field measurements. In this study, in order to examine its sensitivity, the four typical relative permeability functions shown in Figure 6 were applied.
As for the capillary pressure of the soil, i.e. the suction of unsaturated soil and the function of water saturation, two cases are examined, one without capillarity, and another with a certain capillarity as shown in Figure 7.

In actual hydrological processes, evapotranspiration plays an important role in source term evaluation. In this study, a measured average evapotranspiration rate of 3 mm/day was used for reducing rainfall or induced drainage of water from the soil layer.

PARAMETER SENSITIVITY STUDIES

By using short-term observation data, we made simulation runs to check the sensitivity of the parameters, such as Manning’s coefficient, the vertical relative permeability for infiltration, and the soil capillary pressure curve.
Influence of Manning’s coefficient

In this analysis, Manning’s coefficient of surface, except for the riverbed, was changed. A calculated hydrograph is shown in Figure 8. From this result, it is clear that Manning’s coefficient has a great influence on the hydrograph. Additionally, if it is set to a small value (such as 1.0 in this Figure), the variation of flow rate becomes sharper.

As a result, Manning’s coefficient was fixed as large as 20.0 in order to reproduce the relationship of the four peak heights. In existing studies, the equivalent roughness coefficient including riverbed was estimated to be in the range of 5.0–10.0 and surface runoff did not occur in rainfall. In this study, since the roughness coefficient of the riverbed was fixed to 0.04, the authors think that the value of 20.0 for the surface except for the riverbed is appropriate to get the equivalent roughness coefficient described above.

Influence of downward relative permeability of the surface layer

As was mentioned in the previous section, the relative permeability between the surface flow layer and soil layer controls the infiltration of surface water to the subsurface region. In this study, four types of relative permeability table, in contrast to the saturation of the surface flow layer (depth of surface water), were used for the runoff analysis. Figure 9 shows the calculated flow rate for each case. From these results, it can be
seen that the surface relative permeability has a certain influence on the peak values of the calculated hydrograph. In these cases, No. 3 or No. 4 show a relatively good accordance with the measured flow rate in the latter half of the simulated time series.

Influence of capillary pressure of the soil layer

For cases No. 3 and No. 4 in the previous section, the influence of the soil capillary pressure on the hydrograph was analysed. Figure 10 shows the calculated hydrographs. In this Figure, No. 3-b and No. 4-b denote the cases with capillary pressure. In this study, two cases, with and without capillary pressure, were run. It was shown that capillary pressure has a significant influence on the calculated hydrograph for each relative permeability pattern.

In particular, in the former half of the simulated time series, when the soil layer is relatively dry, the calculated hydrograph with capillary pressure shows a better fit with the measured flow rate, compared to the result without capillary pressure. The result shows clearly that consideration of soil capillary pressure is necessary for this type of numerical analysis. The authors also believe that No. 3b shows the best fitting result of all these case studies.

MODIFICATION OF DISCRETIZATION OF THE RIVER BED

Expression of the river bed

In the previous section, the results of several case studies were shown in order to analyse sensitivities of hydraulic properties to the calculated runoff, and to reproduce the measured hydrograph. However, the results show that base flow rate is not reproduced satisfactorily.

To improve the degree of matching, it seemed to be necessary to modify the modelling of the main river channel where the flow was observed continually.

An improved grid system of riverbed is illustrated in Figure 11. In the previous model, surface flow including river flow had been fixed to the second layer of the Finite Difference grid system. With the new grid system, the riverbed is modelled in the third layer (third Layer River Model, 3LR Model), and direct side flows can flow into the river channel, so that the interaction of surface and subsurface flow becomes more realistic.

With this improvement, the following conditions are considered;

(1) Water pouring into the second layer above the river channel flows down quickly to the third (channel) layer.
If water is overflowing from the third layer to the second layer, flow velocities in both layers along the river have the same value.

For the first condition, the permeability between the second and third layers above the riverbed is fixed to a large value, so that in the second layer on the river, the surface flow occurs only when the third layer is flooded to its top. To realize the second condition, horizontal relative permeability in the second layer is formulated as follows.

The flow rates in the third and second layers, respectively, are calculated as

\[
q_3 = \Delta y \Delta z_3 \frac{K_3}{\mu_{w3} B_{w3}} \frac{\partial P_y}{\partial x}
\]

\[
q_2 = \Delta y \Delta z_2 \frac{k_{rw2}}{\mu_{w2} B_{w2}} \frac{\partial P_y}{\partial x}
\]

where, \(\Delta_y, \Delta z\) are the length of the grid in the \(y\)- and the \(z\)-axes, respectively. From the second condition,

\[
q_2 = \frac{\Delta z_2}{\Delta z_3} S_{w2} q_3
\]

From these equations,

\[
\frac{K_3 S_{w3}}{\mu_{w3} B_{w3}} = \frac{K_2 k_{rw2}}{\mu_{w2} B_{w2}}
\]

In the surface flow, since physical properties of water (\(\mu, B\)) can be approximately supposed to be constant, the relative permeability in the second layer has been finally formulated as follows:

\[
k_{rw2} = \frac{K_2}{K_3} S_{w2}
\]

In this numerical simulation, the relative permeability in the second layer is calculated from water saturation (depth) automatically from Equation (12).

**Calculation of hydrograph with the 3LR model**

The authors applied the 3LR model to runo/C128 analysis to the site, in place of the 2LR model. For the first step, hydraulic properties were fixed as those of the optimum 2LR model in the section dealing with
parameter sensitivity studies. The calculated hydrograph is shown in Figure 12. From this result, base flow rates calculated by this model show a good matching with the measured rates, while the peak flow rate and recession curve after rainfall are still different.

In this case, as the riverbed is placed at the bottom of the third layer, the hydraulic properties of the surface or surface soil should be re-considered accordingly. Therefore, the authors modified the parameters of the surface soil in order to make the calculated peaks lower and quicker in the recession part after rainfall. Permeability of the surface soil was changed to $4.7 \times 10^{-1}$ cm/sec, one order of magnitude higher than the previous model, and porosity of surface soil was changed from 0.45 to 0.8. This porosity value seems to be unrealistically large for soil. However, in this model the soil layer includes various types of surface storage effects, so that the authors suppose that it reflects the actual storage potential of the soil layer.

The result of numerical simulation with the above-mentioned hydraulic parameters is shown in Figure 13. In this case, the measured hydrograph seems to be reproduced well. In order to compare the results of these three models, the squared sum of differences between the measured and calculated flow rates with respect to time is shown in Figure 14. From this Figure, it can be seen that the improved model with new hydraulic parameters gives the best result of these three models.

**REPRODUCTION OF ONE-YEAR HYDROGRAPH**

For the purpose of checking if the hydraulic parameters obtained in a short-term history matching can be directly applied to a long-term numerical analysis, the numerical field models obtained in the previous section were applied to the long-term runoff analysis with one-year data shown in Figure 3.

For the long-term analysis, both types of river discretization modelling, i.e. the 2LR and 3LR models, were applied. As precipitation, the average evapotranspiration rate (3 mm/day) was subtracted from the daily rainfall after cutting 10% of rainfall as the intercept caused by the canopies of trees (Suzuki et al., 1979(a),(b)).

Figure 15 shows the calculated result with the 2LR model, and Figure 16 shows the result of the 3LR model. From the result for the 2LR model, the peak flow rates caused by respective rainfalls are overestimated, compared with the measured flow rates. On the other hand, the peak flow rates for the respective rainfalls coincide well with the measured performance from the result of the 3LR model.

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Figure 12. Calculated hydrograph with improved model

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For a more quantitative comparison, the monthly total runoff rate is shown in Figure 17. From this figure, the runoff rate obtained from the 2LR model tends to be greater than the measured runoff. In particular, the calculated and measured runoff rates show a significant difference compared to each other in autumn. On the other hand, the runoff rates obtained from the 3LR model show relatively good accordance to the measured runoff except in June and August.

In 1995, heavy rainfalls occurred in May and July in this region, while relatively little rainfall was observed in June and August. In the numerical simulation with the 3LR model, the flow rate recession in this season shows a significant difference compared to the measured one. The seasonal variance of evapotranspiration appears to be the main reason for this difference. In the numerical simulation, the evapotranspiration value was assumed to be constant through the year because of a lack of observed data. However, this varies according to meteorological conditions. Very small rainfall amounts in August might have reduced the moisture content to a very low level and hence, the evaporation of water from soil might be prevented even when the atmospheric temperature rises. Furthermore, the transpiration from leaves would also be reduced because of the small amount of rainfall in this season.
Figure 15. Calculated long term hydrograph with 2LR model

Figure 16. Calculated long term hydrograph with 3LR model

Figure 17. Measured and calculated monthly runoff
Although there are problems as mentioned above, the improved numerical model has proved its applicability to actual long term simulation in practical use.

CONCLUDING REMARKS

The following points are the main results of this paper.

1. Sensitivity studies on the surface-subsurface interaction parameters were conducted using a short-term hydrograph. As a result, a set of suitable parameters that reproduces the measured short-term hydrograph was obtained.

2. The expression of a river channel in a discretization model was examined using two models. As a result, when setting the surface flow in the second layer and river flow in the third layer, the reproducibility of the simulation was significantly improved.

3. A long-term simulation was carried out to reproduce a hydrograph over a period of one year. The result showed that both short-term and long-term measured hydrographs are reproducible with the same numerical model and same hydraulic parameters.

In addition to the hydrograph, information concerning the groundwater would be of great help in identifying hydraulic parameters and judging the reliability of the field model by a simulator coupled with surface-subsurface flow.

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