Linking genetic algorithms with stochastic dynamic programming to the long-term operation of a multireservoir system

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The objective of this paper is to present a genetic algorithm-based stochastic dynamic programming (GA-based SDP) to cope with the dimensionality problem of a multiple-reservoir system. The joint long-term operation of a parallel reservoir system in the Feitsui and Shihmen reservoirs in northern Taiwan demonstrates the successful application of the proposed GA-based SDP model. Within the case study system it is believed that GA is a useful technique in supporting optimization. Though the employment of GA-based SDP may be time consuming as it proceeds through generation by generation, the model can overcome the “dimensionality curse” in searching solutions. Simulation results show Feitsui’s surplus water can be utilized efficiently to fill Shihmen’s deficit water without affecting Feitsui’s main purpose as Taipei city’s water supply. The optimal joint operation suggests that Feitsui, on average, can provide 650,000 m³/day and 920,000 m³/day to Shihmen during the wet season and dry season, respectively. INDEX TERMS: 1857 Hydrology: Reservoirs (surface); 1884 Hydrology: Water supply; KEYWORDS: genetic algorithms, stochastic dynamic programming, multireservoir system, dimensionality, water supply


1. Introduction

In an area of uneven water distribution, reservoirs can stabilize the water flow to mitigate water problems of flood or drought. Therefore, the task of operating reservoirs efficiently is essential. Both optimization and simulation procedures can be applied to determine the best utilization of reservoirs. However, no single model is available to solve all problems. The choice of techniques to tackle a complex water resources system depends on the characteristics of the specified reservoir system [Yeh, 1985].

Dynamic programming (DP), based on Bellman’s Principle of Optimality [Bellman and Dreyfus, 1962], is commonly used in the optimization of reservoir systems. And stochastic dynamic programming (SDP) is most commonly applied to reservoir systems if the stochasticity of nature is taken into account. Application of SDP to water resource systems has been investigated by many authors [e.g., Su and Deininger, 1974; Alarcon and Marks, 1979; Goulter and Tai, 1985; Wang and Adams, 1986; Huang et al., 1991]. Nevertheless, the applicability of DP/SDP is confined to a few state variables due to the problem of the “curse of dimensionality.” Generally, four state variables with many discrete values may exceed the computational capacity of a computer [Loucks et al., 1981]. Accordingly, many DP-type optimization approaches to cope with the problem of dimensionality in large scale reservoir systems have been introduced [Larson, 1968; Heidari et al., 1971; Turgeon, 1981; Kelman et al., 1990; Saad et al., 1994]. However, a global optimum cannot be verified and only the local optimum is generally obtained [Yeh, 1985].

Genetic algorithms (GA) are a search method based on the principles of Darwinian natural selection and survival of the fittest. GA provides robust procedures to explore broad and promising regions of solutions, and to avoid being trapped at the local optimum. Since 1975, GA has been applied to a variety of fields [Goldberg, 1989]. Several researchers, for instance, Ritzel et al. [1994], Mckinney and Lin [1994], Cieniawski et al. [1995], Milutin and Bogardi [1996], Oliveira and Loucks [1997], and Milutin [1998] have studied the applications of GA to water resource management problems. However, few studies have been done on the application of GA to SDP’s dimensionality problem existing in multireservoir systems of the real world.

As stated by Goldberg [1989], the GA searches from a population of points rather than a single point. Truly, GA compares favorably to the traditional optimization methods due to its robustness in searching solutions. That is, GA works from a rich database of decision variables simultaneously to handle complex problems, such as the dimensionality problem, which are intractable with traditional methods.

In this study, a GA-based SDP is presented to tackle the dimensionality problem of multiple reservoir systems. A real two-parallel-reservoir system, the Feitsui and Shimen reservoir system in northern Taiwan, is selected to demonstrate the applicability of the GA-based SDP model on a complex water resource system.

2. Mathematical Model

For simplicity, let us consider a Y-shaped three-reservoir system as shown in Figure 1. Reservoirs either in series or in parallel exist in the system. Demands $D_i^1$, $D_i^2$, $\ldots$
utilizing the water allocation parameters. In the meantime, the planned water demand of each single reservoir is predetermined by the water allocation parameters. Each single reservoir will be handled independently, and an SDP model with two state variables (storage and inflow) is formulated for each single reservoir.

The key to decomposition of the SDP model is the involvement of GA. The optimal joint operation of the original multireservoir system and optimal water allocation parameter values will be found by combining GA with the individual SDP models. The decision variables are the water allocation parameters in GA, and the fitness function is the integrated sum of each individual SDP objective value. Namely, GA searches for the optimal water allocation parameter values so that the overall integrated function is optimized. A flowchart of the GA-based SDP is given in Figure 2, and the procedure of SDP by applying GA to a multiple-reservoir system is interpreted in detail as follows.

1. Define a chromosome representing a set of release allocation parameter \( a_i^j \), for example, \( (a_{11}^i, a_{23}^i, a_{14}^i, a_{24}^i, a_{34}^i) \). Note that the parameters, \( a_{11}^i \) and \( a_{23}^i \), are fixed to be one here.

2. Determine the planned water release of the individual reservoirs. For example, see Figure 1. shows the request demands (targets) on reservoirs 1, 2, and 3 are, respectively, \( (a_{11}^i D_1^i + a_{13}^i D_3^i + a_{14}^i D_4^i), (a_{23}^i D_2^i + a_{24}^i D_4^i), \) and \( (a_{34}^i D_3^i + a_{24}^i D_4^i) \).

3. Derive an SDP-based operating policy for each single reservoir, and the sum of the individual SDP objective values yields the overall performance of the multiple-reservoir system.

4. Repeat steps 1 to 3 for different chromosomes within a specified generation. Then each individual in a population can be evaluated in terms of the fitness function.

5. Employ GA essential operators (selection, crossover, and mutation) to create a new generation from the present generation given in step 4.

6. Return back to step 1 to reassess the new generation until the termination criterion, either the maximum number of generations or the convergence of average population fitness, is met. Then, a chromosome with the best fitness value is selected as the optimal water allocation parameter \( a_i^j \), and the corresponding joint operating policies of a multiple-reservoir system are finally found.

[11] In step 1, each parameter is expressed by a binary string of length \( N \). Initially, the chromosome is generated at random within binary-coded GA. The linear mapping relationship between a binary number and the associated real value of \( a_i^j \) is

\[ a_i^j = a_{\text{min}} + \left( \frac{a_{\text{max}} - a_{\text{min}}}{2^N - 1} \right) \sum_{n=1}^{N} \left( 2^{n-1} b_n \right) \]  

where the interval \( (a_{\text{min}}, a_{\text{max}}) \) shows the minimum and maximum values of parameter, \( a_i^j \) and \( b_n \) indicates a value of 1 or 0 inside the binary string. Since the interval of water allocation parameter value equals \((1, 0)\), equation 4 then becomes

\[ a_i^j = \sum_{n=1}^{N} \left( 2^{n-1} b_n \right) \left( \frac{2^N - 1}{2^N - 1} \right) \]
[12] In the situation of step 3, assuming no significant seepage and evaporation losses, the release of reservoir 3 and the excess releases of reservoir 1 and 2 in period t, respectively, equal

\[ R_3^t = S_3^{kt} + Q_3^{it} / C_0 S_3^{l, t+1} + X_1^t + X_2^t \]

\[ X_1^t = \max \{0, R_1^t - (a_1^{11} D_1^t + a_1^{13} D_1^t + a_1^{14} D_1^t)\} \]

\[ X_2^t = \max \{0, R_2^t - (a_2^{22} D_2^t + a_2^{23} D_2^t + a_2^{24} D_2^t)\} \]

[13] Obviously, the SDP optimization can be independent if all reservoirs are in parallel and release targets for each reservoir, as mentioned in step 2, are predetermined. However, for a serially connected multireservoir system, the excess releases from the upstream reservoirs will become additional inflows to the downstream reservoir. Inflows of the downstream reservoirs are related to the inflows of the upstream reservoirs. Certainly, the traditional joint SDP optimization by considering the joint probability of inflows for serial connection offers one solution. However, the dimensionality problem subsequently arises. It is quite clear that once the excess releases from upstream reservoirs are given, there is no difficulty when deriving the optimal SDP policies for the downstream reservoir alone. Unfortunately, the quantities of the excess releases are random and dependent on a function of the upstream reservoir's storage and inflow. We have to consider the correlation of inflows when decomposing a serial reservoir system. However, it is a challenging task to estimate the possible amount of excess releases of upstream reservoirs when considering the inflow relationship. Further, when deriving the optimal SDP policies for reservoir 3, the optimal SDP policies for reservoir 1 and reservoir 2 must be obtained in advance. The procedure to estimate the possible excess is presented below.

[14] As is known for optimal SDP policies of reservoir j, a unique release \( *R_{kt}^{ij} \) is optimal for each \( S_{kt}^{ij} \) and \( Q_{it}^{ij} \), i.e., \( *R_{kt}^{ij} = f(S_{kt}^{ij}, Q_{it}^{ij}) \) for all k and i. Subsequently, the excess release can be determined in terms of its required release target. Clearly, the determined excess is associated with the joint probability of \( P(S_{kt}^{ij}, Q_{it}^{ij}) \) for any value of \( k, i, \) and \( t \). On the basis of optimal policies, the steady state joint proba-
probabilities of \( P(S^t_l, Q^t_i) \) can be attained by solving the following equations

\[
P(S^t_l, Q^t_i) = \sum_k \sum_j P(S^t_l, Q^t_j) = \sum_k \sum_j P(Q^t_i | Q^t_j) \quad \text{for all } i, h, t
\]

(9)

\[
\sum_k \sum_j P(S^t_l, Q^t_i) = 1.0 \quad \text{for all } t
\]

(10)

where \( P(Q^t_i | Q^t_j) \) indicates the inflow transition probabilities of \( j \)th reservoir between period \( (t+1) \) and period \( t \). In our example, as inflow is in state \( i \), the expected excess release of reservoirs 1 and 2 are

\[
Y^1_k = \sum_i P(S^t_i, Q^t_i)
\]

* \[ \max \{0, R^1_k - (a_{1i} D^1_t + a_{2i} D^2_t + a_{3i} D^3_t + a_{4i} D^4_t) \} \quad \text{for all } i \]

(11)

and

\[
Y^2_k = \sum_i P(S^t_i, Q^t_i)
\]

* \[ \max \{0, R^2_k - (a_{2i} D^2_t + a_{3i} D^3_t + a_{4i} D^4_t) \} \quad \text{for all } i \]

(12)

[15] Further, let \( P(Q^t_j | Q^t_i) \) represent the inflow transition probabilities specifying the conditional probability of \( Q^t_j \) given \( Q^t_i \) between serial connected reservoirs. Thus the expected excess releases over inflow states of reservoirs 1 and 2 are equal to

\[
E(Y^1_j) = \sum_i P(Q^t_i | Q^t_j) Y^1_i \quad \text{for all } j
\]

(13)

\[
E(Y^2_j) = \sum_i P(Q^t_i | Q^t_j) Y^2_i \quad \text{for all } j
\]

(14)

[16] Once the expected excess releases are attained, we can derive the SDP optimal policies of the serial reservoir 3 in terms of the following continuity requirement

\[
R^i_{kt} = S^t_i + Q^t_j - S^t_{k+1} + E(Y^i_j + X^2_j) \quad \text{for all } k, j, t
\]

(15)

[17] On the other hand, chromosomes of the current population are selected through the application of biased roulette wheel selection [Goldberg, 1989]. The probability of selection of a chromosome is proportional to its “weight”, defined by individual fitness value over all individuals in the population. Clearly, a chromosome with better fitness has a higher probability of being selected. Once a pair of parent chromosomes is picked, the crossover operator is used to randomly choose crossover site(s) along parent strings to reproduce two new chromosomes as children. In addition to selection and crossover operators, mutation changes the genes of chromosomes at a very low incidence of chance, but gives diversity within a population. It changes the bit value from 1 to 0 and vice versa by selecting mutation point(s) randomly along children strings. Thereafter a new generation arises.

[18] Though the GA works powerfully in search, it works with a coding of the decision variable set rather than the decision variables themselves [Goldberg, 1989]. Problems related to infeasible solutions may occur when constraints exist. Since the water allocation parameters \( (a^1, a^2, a^3, a^4, a^5, a^6) \) inside a chromosome are interdependent with \( a^1 + a^2 = 1 \) and \( a^1 + a^3 + a^4 = 1 \), infeasible solutions through GA operators may occur. So an adjustment for infeasible chromosomes is needed. J. J. Bogardi (personal communication, 1999) suggested applying a repair procedure. According to equation 5, we have \( \alpha^j_l = \alpha^j_l/(2^N - 1) \) where \( \alpha^j_l = \sum_{n=1}^{N_l} (2^{n-1} b_{n_l}) \). Since \( \sum \alpha^j_l = 1 \), we have

\[
\sum \alpha^j_l = (2^N - 1) \quad \text{for all } j
\]

(16)

[19] Equation 16 indicates an integer representation for the required coding of feasible solutions. The case on \( \sum \alpha^j_l > (2^N - 1) \) results in an overestimate of parameters, and underestimate for \( \sum \alpha^j_l < (2^N - 1) \). We may repeatedly select \( \alpha^j_l \) at random from the infeasibles subset to adjust its binary values until the condition in equation 16 is satisfied.

3. Case Study

3.1. Statement of the Problem

Taiwan has abundant rainfall of about 2,510 mm/yr. However, the precipitation distribution is very uneven, with 78% coming during the wet season (May through October) and only 22% during the dry season (November through April). Due to the uneven surface runoff, reservoirs are widely used to retain excess water for drought mitigation. At present, the total water withdrawal is approximately 18 billion cubic meters per year. Reservoirs play an important role during the dry period. Currently, about 25% of annual water use comes from reservoir supply.
3.2. SDP Formulation

[21] The case study system shown in Figure 3 has two reservoirs (Feitsui and Shihmen) in parallel. These two reservoirs are the largest and most important in northern Taiwan. The proposed methodology can readily handle the parallel connection. In the meantime, the cases of serial connection deserve further study using the proposed method.

[22] The Feitsui reservoir was completed in June 1987. The effective reservoir capacity is 359 million cubic meters (MCM) along with a catchment area of 303 km². The reservoir is located in the upstream of the Hsintien River, 30 km southeast of Taipei. The primary purpose of Feitsui reservoir is water supply for the people of metropolitan Taipei. The 2001 water demand was 1,172.43 MCM/yr. It is expected that, with the help of this reservoir, the estimated water demand of the Taipei metropolitan area can be satisfied adequately until 2030. The Shihmen reservoir was built in 1963 with effective capacity of only 234 MCM. It has a larger catchment area (754 km²) than the Feitsui does. The total demand dependent upon the Shihmen reservoir was 1,163.59 MCM/yr in 2001. Of this, 529.09 MCM/yr is used for irrigation over the Taoyuan area, and public water supply takes 634.50 MCM/yr for the people living in nearby Taipei city. At present, these two reservoirs are operated separately by different agencies, and problems of public water supply deficit frequently arise in Shihmen’s operation while Feitsui has much surplus water lacking efficient utilization. Therefore, joint optimal operation between the two parallel systems is necessary. Currently, a channel is available to transport water from the Chihtan water treatment plant of Feitsui to the Banhsin plant of Shihmen. Further, in order not to detract from Feitsui’s original system operation for Taipei city, the additional water to the Banhsin plant should come from Feitsui’s surplus water. The GA-based SDP seems suitable to solve this pressing operational problem. Details of the feasibility of joint operations between reservoirs by use of GA-based SDP will be discussed in this paper.

### 3.2. SDP Formulation

[23] The reservoir system in Figure 3 is similar to that in Figure 1, except there is no serial connection to a downstream reservoir. For a joint SDP-based model in this system, five state variables (two storages and three inflows) exist.

[24] First of all, with respect to the discretization schemes here, the storage class intervals are defined subjectively to be the same with one MCM. Therefore Feitsui’s and Shihmen’s effective storages will be divided into 36 and 24 classes respectively. On the other hand, the class intervals of inflow will be defined with same expected number of observations, and the interval widths will become unequal. Subsequently, the number of representative discrete values of streamflow for Feitsui, the tributary (supplementary discharge relevant to meet the water demand downstream), and Shihmen are set to 9, 9, and 20 respectively. In the case of joint SDP in this study, the number of possible system state transitions to be explored at each stage will be $(9*9*36)*(20*24)*(36*24) = 1,209,323,520$. In this study, a 10-day-long flow is adopted as the basis for analyzing the SDP-based optimization in each period. There are 36 stages within a year. Table 1 illustrates the possible system state transition related to SDP calculations within a stage. Obviously, the computation work becomes tedious and the dimensionality problem will be encountered for a fine discretization of storage and inflow. Thus a single SDP-based model for each reservoir is considered instead, as described in detail below.

#### 3.2.1. Objective Function

[25] Since the impact of water deficit to the multireservoir system is the primary concern here, the objective is set to minimize expected (annual) demand deficit, i.e.,

$$\text{Minimize } E[\min(0, R_t - T_t)]^2$$  \hspace{1cm} (17)

where $R_t$ and $T_t$ are, respectively, water release including spillage and target at period $t$. Similar objective forms were used for Feitsui and Shihmen.

#### 3.2.2. Constraints

$$S_{t+1} = S_t + Q_t - R_t - L_t$$

$$= \max[0, \min\{S_t + Q_t - R_t - L_t, K\}] \text{ for all } t$$  \hspace{1cm} (18)

where $S_t$, $Q_t$, and $L_t$ are the initial reservoir storage volume at the beginning of period $t$, the inflow during that period, and the evaporation loss from the reservoir respectively. $K$ indicates the reservoir capacity.

#### 3.2.3. Recursive Equation

[26] Let $f^n(S_t, Q_t)$ be the total expected value of the system performance with $n$ periods to go, associated with the state variable vector. Then the backward recursive relationships can be generalized by

$$f^n(S_t, Q_t) = \text{Minimize } \left\{ [\min(0, R_t - T_t)]^2 \right. $$

$$+ \sum P_{Q(t+1)|Q(t)} * f^{n-1}(S_{t+1}, Q_{t+1}) \}$$  \hspace{1cm} (19)

where $P_{Q(t+1)|Q(t)}$ defines the inflow conditional probability that the next inflow state in period $t + 1$ is at state $Q(t + 1)$, given the current inflow in period $t$ is at state $Q(t)$.

[27] Generally, steady state policy can be reached over a few annual cycle computations by using backward-moving SDP [Huang et al., 1991]. During the recursive calculations of SDP, however, the recursive values cannot be increased, i.e., the expected system performance in each cycle could be decayed to zero, if the elements in row of the transition probability matrix are overall zero. Accordingly, the steady state operating policy derived from the solution of the SDP as a function of the initial storage volumes and inflows will never be attained under these conditions. The undesired

<table>
<thead>
<tr>
<th>Initial State</th>
<th>Final State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(S_t^i, Q_t^j)$</td>
<td>$(S_{t+1}^i, S_{t+1}^j)$</td>
</tr>
<tr>
<td>Number of classes = $(24<em>20)</em>(36<em>9</em>9)$</td>
<td>No. of classes = $(24*36)$</td>
</tr>
<tr>
<td>System dimension $= (24<em>20)</em>(36<em>9</em>9)<em>(24</em>36)$</td>
<td>$= 1,209,323,520$</td>
</tr>
</tbody>
</table>

### Table 1. System State Transition of SDP Within a Stage
occurrence of this condition may be avoided either by careful discretization of reservoir inflows or by synthetic inflow generation [Huang and Wu, 1993]. Since the number of discrete values of streamflow for Feitsui, the tributary, and Shihmen are, respectively, equal to 9, 9, and 20, the dimension of transition matrix for Feitsui becomes (9*9)*(9*9), and Shihmen’s (20*20). They are especially large as compared with the historical streamflow data (1954–1994). In this paper, an AR(1) model is adopted to generate sufficient synthetic inflows. Further, the travel time of water within the system can be ignored, as compared with the 10-day-long period used here.

### 3.3. GA Formulation

[28] As seen in Figure 3, the water quantity to the Banhsin plant is delivered jointly from the Feitsui and Shihmen reservoirs. The parameters, (aw, F, ad) and (aw, S), indicate percentages of release allocation from Feitsui and Shihmen during the wet and dry seasons respectively. With aw + ad = 1 and ad + ad = 1 as constraints, only aw and ad are selected as variables inside a chromosome in this study. Thus infeasible solutions to violate the constraints above can be avoided within the GA runs. The adopted chromosome includes two 8-bit sub-strings representing aw and ad separately.

[29] The GA search begins with a randomly generated population of strings. Each 8-bit-long chromosome shows potential solutions to the SDP-based problem. In this paper, a classic GA with proportional selection, one-point crossover, and single-bit mutation is applied. In addition, there are 50 individuals in a generation and a maximum of 70 generations, and crossover and mutation probabilities are assigned as 0.8 and 0.01, respectively. Since the objective function of the single SDP model is to minimize the squared deficit of demands over a year, the GA fitness function is set as follows.

\[
\text{Minimize } E\left[\min(0, R_i F - T_i F)\right]^2 + E\left[\min(0, R_i S - T_i S)\right]^2
\]

[30] Following the flowchart in Figure 2, an optimal joint operation of GA-based SDP between Feitsui and Shihmen reservoirs can subsequently be derived.

### 3.4. Results Analysis

#### 3.4.1. Independent Operation

[31] For the case of independent reservoir operations by Shihmen and Feitsui, i.e., (aw = 1, ad = 1) and (aw = 0, ad = 0), the individual simulation outcomes (1954–1994), based on derived independently optimal SDP operating policies, indicate inefficient water utilization within the system (see Tables 2 and 3). In this study, simulation starts at full storage of each reservoir, and some practical indicators are used to evaluate the system performance. That is, (1) shortage index (SI), a general shortage index [U.S. Army Corps of Engineers, 1963] is defined as

\[
SI = \frac{100}{n} \sum_{i=1}^{n} \left( \frac{S_a}{D_a} \right)^2
\]

(2) average shortage ratio (ASR),

\[
ASR = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{S_a}{D_a} \right)
\]

(3) average shortage period (ASP),

\[
ASP = \sum_{i=1}^{n} \left( \frac{S_a}{n} \right)
\]

where Sa is annual shortage (annual demand-annual supply), Da is annual demand, n is number of years considered, and

### Table 2. Simulation Results of Shihmen Reservoir System (1954–1994)

<table>
<thead>
<tr>
<th>Allocation parameters</th>
<th>Single-Reservoir Operation</th>
<th>Multireservoir Operation a</th>
<th>Multireservoir Operation b</th>
</tr>
</thead>
<tbody>
<tr>
<td>da = 1, aw = 1</td>
<td>1163.59</td>
<td>0.8078, aw = 0.338</td>
<td>0.078, aw = 0.338</td>
</tr>
<tr>
<td>SI</td>
<td>4.145</td>
<td>0.361</td>
<td>0.323</td>
</tr>
<tr>
<td>ASR(%)</td>
<td>17.69</td>
<td>5.03</td>
<td>4.45</td>
</tr>
<tr>
<td>ASP (period/year)</td>
<td>20.71</td>
<td>7.46</td>
<td>9.37</td>
</tr>
<tr>
<td>FS(%)</td>
<td>42.48</td>
<td>79.27</td>
<td>73.98</td>
</tr>
</tbody>
</table>

a Including the quantity delivered to Shihmen reservoir system.
b Excluding the quantity delivered to Shihmen reservoir system.

### Table 3. Simulation Results of Feitsui Reservoir System (1954–1994)

<table>
<thead>
<tr>
<th>Allocation parameters</th>
<th>Single-Reservoir Operation</th>
<th>Multireservoir Operation a</th>
<th>Multireservoir Operation b</th>
</tr>
</thead>
<tbody>
<tr>
<td>da = 0, aw = 0</td>
<td>1172.43</td>
<td>0.922, aw = 0.662</td>
<td>0.922, aw = 0.662</td>
</tr>
<tr>
<td>Annual demand (MCM)</td>
<td>1172.43</td>
<td>1467.99</td>
<td>1172.43</td>
</tr>
<tr>
<td>SI</td>
<td>0.023</td>
<td>0.093</td>
<td>0.037</td>
</tr>
<tr>
<td>ASR(%)</td>
<td>0.40</td>
<td>1.40</td>
<td>0.76</td>
</tr>
<tr>
<td>ASP (period/year)</td>
<td>0.32</td>
<td>2.63</td>
<td>0.78</td>
</tr>
<tr>
<td>FS(%)</td>
<td>99.12</td>
<td>92.68</td>
<td>97.83</td>
</tr>
</tbody>
</table>

a Including the quantity delivered to Shihmen reservoir system.
b Excluding the quantity delivered to Shihmen reservoir system.
S_p is annual shortage period; and (4) frequency of success (FS), Prob(S_a < 0) shows the percent of time that supply can meet demand.

[32] For the Shihmen reservoir, the annual demand is 1,163.59 MCM, including 373.248 MCM for public water supply at the Banhsin plant. Simulation results indicate the current demand can be fulfilled 42.48% of the time and SI = 4.145. Water deficiency events occur every year. The average deficit is about 5.72 MCM per 10-day period, and the year 1973 is the driest one with a total shortage of 465.5 MCM. The average number of shortage periods is approximately 20.71 period/yr, and the average shortage ratio of annual shortage to annual demand is up to 17.69%. Overall, Shihmen’s operational performance is poor. This may reveal that demand for water on Shihmen is a little large at 1,163.59 MCM/yr. In the meantime, about 282.86 MCM is spilled annually. As compared with the abundant inflow to the Shihmen reservoir (1,393.473 MCM/yr), Shihmen’s effective capacity (234 MCM) is relatively small. Of course, the severity of water deficiency could be softened by enlarging Shihmen’s capacity to save spilled water. To alleviate the problem, the government has planned to build an upstream reservoir (Gaotai reservoir). But in the meantime, the shortages must be lessened with the help of other sources, and one of the sources would be the neighboring Feitsui reservoir.

[33] For the Feitsui reservoir, the annual demand is 1,172.43 MCM. The current demand can be satisfied with SI = 0.023, as the other evaluation indicators of ASR, ASP and FS, respectively, equal 0.4%, 0.32 period/yr, and 99.12%. Normally, a designated SI value for water supply planning in Taiwan is set to be one, and SI = 1 is regarded as the severity level of 10% shortage every year. Hence SI = 0.023 can fully satisfy the essential criterion. In addition, droughts only occur in 1955, 1963, 1977, and 1994 during the simulation period, and the occurrence in 1963 is the most serious drought in history. The total shortage in 1963 reaches 83.88 MCM. Obviously, Feitsui’s excellent performance can reach the target to fulfill Taipei’s demand as planned. Meanwhile, water quantities of 1,112.42 MCM on average are annually released to the downstream without efficient utilization. The surplus water might be available to fill additional demand, such as supplementation to Shihmen. Feitsui’s surplus could play an important part to offset Shihmen’s deficit. As discussed, joint SDP-based optimal operation between them is needed to balance their water utilization.

3.4.2. Joint Operation with GA

[34] On the basis of GA formulation mentioned above, Figure 4 depicts the variation of the best and average fitness value of each generation, separately. It was found that the best fitness might perform a little worse than the preceding generation due to mutation. However, average performance of the GA-based SDP system approaches steadiness after 30 generations. Further, the required computation time using a Pentium III 1 GHz processor for the optimal SDP search for the individual chromosome is as much as 128 hours through 70 generations. Although time-consuming, this process avoids the problem of dimensionality in optimization. The optimal release allocation parameters, (\(a^w_0 = 0.662, a^d_0 = 0.922\)) and (\(a^w_0 = 0.338, a^d_0 = 0.078\)) in the fifteenth generation are selected as the best ones of water supply ratio for SDP modeling. That is, the Banhsin plant depends upon Feitsui to satisfy 66.2% and 92.2% of its demand (1.0368 MCM/day) during the wet and dry seasons respectively, and on Shihmen for the remaining 33.8% and 7.8%. Total additional demand met by Feitsui is up to 295.56 MCM per year. Under these conditions, the demand on Shihmen has dropped from 1,163.59 MCM/yr to 868.03 MCM/yr.

[35] Based on the optimal joint operation of GA-based SDP derived policies, simulation results in Table 2 show that Shihmen’s water deficit has been alleviated significantly by Feitsui’s surplus water. Joint operation for Shihmen is obviously more advantageous than the independent operations mentioned above. With the same basis of demand (1,163.59 MCM/yr), for instance, the evaluation indicators (SI, ASR, ASP, FS) of Shihmen’s operating status have been improved from (4.145, 17.69%, 20.71, 42.48%) to (0.323, 4.45%, 9.37, 73.98%) after obtaining Feitsui’s assistance. Though 37 deficit years still exist within the simulation, the average shortage per period has fallen dramatically to 1.3 MCM from 5.72 MCM, and the shortage in 1973, the most serious drought year, has dropped to 168.39 MCM from as high as 465.5 MCM.

[36] The burden of water supply on Shihmen reservoir will become lighter with lower demand at 868.03 MCM/yr. The performance corresponding to (0.361, 5.03%, 7.46, 79.27%) indicates the joint operation has considerably reduced the pressure of water supply on Shihmen. Subject
to the constraint of giving priority to satisfying Taipei’s demand, the operation on Shihmen is still satisfactory. But the time in which demand is met is still short (79.27%); that is, shortage events occur frequently here. It seems serious water deficiency would be unavoidable if the water demand on Shihmen continues to rise in the future. Thus, in addition to Feitsui, it is necessary for the Taiwan Water Supply Corporation (TWSC) to buy extra water from other sources such as the agricultural sector at a fee when the water supply demand cannot be met. The TWSC could pump ground-water in case of emergency, but this option is negated by restrictions on the use of groundwater in order to prevent land subsidence.

Given that Feitsui must fulfill Taipei’s demand with first priority before exporting surplus water to the Banhsin plant, the joint operation should not weaken Feitsui’s original function of serving Taipei city. As far as Taipei’s water supply is concerned, the results through simulation (see Table 3) with identical demand (1,172.43 MCM/yr) give values of (0.037, 0.76%, 0.78, 97.83%). As compared with the performance indices (0.023, 0.40%, 0.32, 99.12%) of independent operation, the negative impact of joint operation to Feitsui appears insignificant and allowable. Though 11 deficit years exist through the simulation for Taipei’s water supply, the severity of water shortage did not worsen. For example, the deficit water in the driest year (1963) is 87.66 MCM, close to the quantity of 83.88 MCM in the independent operation.

When supporting Shihmen, however, Feitsui’s overall performance, (0.093, 1.40%, 2.63, 92.68%), does become a little inferior to its performance in independent operation. This is because the request demand to Feitsui has increased from 1,172.43 MCM/yr to 1,467.99 MCM/yr, where the extra demand (295.56 MCM/yr) for the Banhsin plant is included. Undoubtedly a deficit event will occur at the Banhsin plant when Feitsui cannot provide sufficient water to Taipei. In all, two-thirds of the simulation periods (27 years) encounter water shortage at the Banhsin plant. But the average deficit duration is not long and the average shortage quantity is not large. The average delivery quantity to the Banhsin plant equals 283.91 MCM/yr. As shown in Figure 5, on average, the reliability of Feitsui to fulfill the water request of Banhsin plant is above 85% in each period, and the delivery quantities from Feitsui to the Banhsin plant are about 920,000 m$^3$/day (CMD) during the dry season and 650,000 CMD during the wet season. The lack of water ranges between 0.15 and 99.87 MCM/yr, with 99.87 MCM appearing in 1963. Furthermore, after adopting joint operation, the average surplus water released downstream of Feitsui without being utilized has been reduced to 833.70 MCM/yr from 1,112.42 MCM/yr. The difference reflects the effective application of Feitsui’s surplus water to supplement Shihmen.

It is noted that the indicator of the average shortage ratio could distort the calculation of system performance if the denominator in equation 22 were m (number of years where deficit occurs), instead of n (number of years simulated). The former represents the average shortage ratio where shortage years occur. For Feitsui’s operation, for example, m equals 4 and 11, respectively, for independent and joint operations. Accordingly, on the basis of Taipei’s demand (1,172.43 MCM/yr), the ASR would be 4.10% and 2.83% respectively. Using “m” in equation 22 would probably cause a misleading assessment as m is not large enough in statistical sampling. Instead, 0.40% and 0.76%, respectively, are more appropriate in terms of n = 41 in this study.

The existence of the dimensionality problem in SDP for multireservoir systems inhibits the identification of a global optimum of SDP operating policy. Therefore, in order to explore a broad spectrum of potential solutions, it is necessary to start the GA search from a randomly generated population of strings. In this study, the GA-based SDP model has been run ten times with different initial
random populations to examine the statistical characteristics of selected parameters of chromosome. Figure 6 shows the variation of the best selected parameters in runs. Since the sample size (10 runs only) is small, and the population variance is unknown, we can use t-distribution to determine the corresponding upper and lower confidence limits [Ang and Tang, 1975]. On this basis, the appropriate 99% confidence interval for the population mean of $a_1$ and $a_2$ equal (0.334; 0.385) and (0.061; 0.087), respectively. The differences are small, and they assure us that GA-based SDP can search and approach a possible global optimum for a multireservoir system.

4. Conclusions

The “curse of dimensionality” of stochastic dynamic programming (SDP) can be resolved through genetic algorithms (GA). The GA-based SDP model introduces: (1) decomposition of a multireservoir system into a set of single-reservoir subsystems; (2) formulation of potential release allocation for each water user within the system by use of GA; (3) optimization of operating policy for each single-reservoir by use of SDP. In this paper, the GA-based SDP model is applied to a real-world two-parallel-reservoir water supply system in northern Taiwan. Within the case study system, the joining of GA and SDP well demonstrates that optimal joint long-term operating policies between the Feitsui and Shimen reservoirs can be achieved while avoiding the problem of dimensionality. On the other hand, for a serially connected multireservoir system, the proposed methodology here deserves further study.

In terms of release allocation parameters as variables in chromosomes of genetic algorithms, the “global” optimum of SDP can be obtained through generations. A classic GA with proportional selection, one-point crossover, and single-bit mutation was used here. Results show the GA-based SDP model approaches steadiness after 30 generations, where 50 individuals in a generation and the maximum of 70 generations were taken into account. Truly, GA is a sort of trial and error method in searching optimal solutions, and it may be time-consuming in generation by generation searching involving selection, crossover and mutation. However, it avoids the “dimensionality curse” in searching solutions, and the computation time can also be reduced further with powerful computers or parallel processing.

Obviously, GA is a very efficient tool in supporting an SDP-based multireservoir system as demonstrated in this study. On the basis of the derived GA-based SDP operating policies for Feitsui and Shihmen, the simulation results (1954–1994) show the superiority of joint operation over independent operation. Feitsui’s surplus water can fill Shihmen’s deficit water without affecting Feitsui’s main function as Taipei city’s water supply. On average, Feitsui can supply Shihmen with 650,000 m$^3$/day and 920,000 m$^3$/day during the wet season and dry season, respectively. The reliability in each period of meeting Shihmen’s water request from Feitsui is above 85%.

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