Hydraulic Design of Stepped Spillways

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Abstract: An experimental study on a large model flume using fiber-optical instrumentation indicated that the onset of skimming flow is a function of critical depth, chute angle, and step height. Uniform mixture depths that determine the height of chute sidewalls and uniform equivalent clear water depths are described in terms of a roughness Froude number containing unit discharge, chute angle and step height. The spillway length needed to attain uniform flow is expressed as a function of critical depth and chute angle. The flow resistance of stepped spillways is significantly larger than for smooth chutes due to the macro roughness of the steps. The friction factor for uniform aerated flow is of the order of 0.1 for typical gravity dam and embankment dam slopes, whereas the effect of relative roughness is rather small. The energy dissipation characteristics of stepped spillways and the design of training walls are also discussed. The paper aims to focus on significant findings of a research program and develops design guidance to lessen the need for individual physical model studies. A design example is further presented.

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Introduction

Thanks to the technological advances in construction of roller compacted concrete (RCC) dams over the past 2 decades, stepped spillways for discharging excess flood water have gained significant interest and popularity among researchers and dam engineers, both for new dams and for armoring of existing embankment dams. The use of stepped spillways has enhanced the performance and economy of many RCC dams where the concrete placement in lifts allows an economic and fast construction of spillway steps on the downstream dam faces.

The main advantage of stepped chutes over conventional smooth spillways, in addition to construction economy, is the significant energy dissipation along the chute due to the macro roughness of the steps. This in turn leads to a pronounced reduction of the stilling basin dimensions at the toe of the spillway. Cavitation risk resulting from excessive subatmospheric pressures diminishes due to greatly reduced flow velocities and the high amount of air entrainment. The aeration produces flow bulking, however, and therefore requires higher sidewalls.

Few definite design guidelines for stepped spillways are available to date. To fill in this gap, an experimental model study on the characteristic two-phase skimming flow down a stepped chute was conducted at the Laboratory of Hydraulics, Hydrology and Glaciology (VAW) of the Swiss Federal Institute of Technology (ETH), Zurich (Boes 2000; Schläpfer 2000). Besides the investigation of scale effects, the inception of air entrainment, air concentration and velocity distributions (Boes and Hager 2003), the prediction of the onset of skimming flow, uniform flow depths and distances to attain uniform flow, friction factors and the energy dissipation characteristics for skimming flow on stepped chutes are dealt with in the following.

Experimental Setup

All experiments were conducted in a prismatic rectangular channel of width b = 0.50 m and length L = 5.7 m with bottom angles φ from the horizontal of 30°, 40°, and 50° or slopes (V:H) of 1:1.73, 1:1.19, and 1:0.84 (Fig. 1). Three step heights s = 23.1, 46.2, and 92.4 mm were investigated for the 30° cascade, steps of 31.1 and 93.3 mm were tested for the 50° chute and of 26.1 mm for φ = 40°. Only the spillway face with constant bottom slope and step size was considered. A so-called jetbox transformed the pressurized approach flow to a free surface open channel flow of precalibrated approach flow depth and approach velocity. Despite this arrangement, all results presented hereafter refer to standard ungated stepped spillways, because the fictitious location of the spillway crest was deduced analytically from general drawdown equations (Hager and Boes 2000), which agrees well with data from crest profile spillways (Boes and Hager 2003).

A two-tip fiber-optical probe was used to measure both air concentrations and flow velocities in selected outer step edge cross sections. This instrumentation is described by Boes and Hager (1998) and Boes (2000), its physical measuring principal being based upon the different optical refraction indices of air and water. A detailed description of the experimental setup is given by Boes (2000) and Boes and Hager (2003).

Transition from Nappe to Skimming Flow

Two distinct flow regimes occur on stepped spillways, so-called nappe flow and skimming flow. Whereas in nappe flow the steps...
act as a series of overfalls with the water plunging from one step to another (Fig. 2), the water flows as a coherent stream over the pseudobottom formed by the outer step edges in skimming flow, without air pockets under the jets. This definition has been adopted by most researchers (see Matos 2001; Ohtsu et al. 2001). Generally speaking, nappe flow is found for low discharges and large steps. For small steps and large discharges such as the design discharge the water usually skims over the step edges, and recirculating zones develop in the triangular niches formed by the step faces and the pseudobottom (Fig. 3). Strictly speaking, a distinction between an upper limit for nappe flow and a lower limit for skimming flow may be defined, with a transition regime separating these characteristic limits. The transition from nappe to skimming flow can be expressed by the ratio of critical depth \( h_c \) and step height \( s \). The present experimental results indicate the onset of skimming flow at (Fig. 4)

\[
\frac{h_c}{s} = 0.91 - 0.14 \tan \phi
\]  

This is in agreement with the onset/transition functions and data of Rajaratnam (1990); Stephenson (1991); Chanson (1996); Tatewar and Ingle (1999); Yasuda and Ohtsu (1999); and Matos (2001) for approximately 25°<\(\phi\)<55°. Functions proposed by Chamani and Rajaratnam (1999a) and James (2001) are likely to underestimate the transition from nappe to skimming flow, which might be caused by hydrodynamic instabilities resulting from a change from aerated to unaerated nappes in the transition regime (Chanson 1994), the flow should be distinctly either in the nappe flow or the skimming flow regime for the design discharge and the safety check flood. A comparison of the results of Yasuda and
Ohtsu (1999) with Eq. (1) results in about 20% of deviation from the $h_c/s$ values. Obviously, for ungated spillways, the transition regime cannot be avoided if the chute is designed for skimming flow.

**Uniform Two-Phase Flow**

Regarding the attainment of uniform flow towards the downstream spillway end, all experimental runs were basically evaluated in three different ways. First, the air concentration profiles at the chute end and at a cross section further upstream were compared. Agreement of the two as in Fig. 5 was considered sufficient for fully developed two-phase flow with quasiconstant aeration rate, flow depth and velocity. For the majority of the present experimental runs, this was attained at the spillway end. However, observations of Matos et al. (1999) and Matos (2000a) indicated that uniform flow may not have been fully developed according to this criterion. Therefore, a second method to check uniform flow conditions consisted in examining the drawdown curves described by the equivalent clear water (subscript w) and characteristic mixture depths $h_w$ and $h_{90,w}$, respectively. For quasiconstant values at the downstream spillway end, uniform flow was likely to be attained. Drawdown curves which had not yet reached their uniform (subscript u) depth were extrapolated to obtain $h_{w,u}$ and $h_{90,u}$ (Fig. 6). Accordingly, experimental runs where uniform flow had not yet been fully attained at the chute end could also be retained for subsequent analysis. A third criterion was further applied for the computation of friction factors where $h_{w,u}$ enters in the third power so that even slight deviations may have a significant effect on the friction factor value, see Eq. (14). Because $C_u$ is similar both for smooth and rough chutes (Matos 2000b; Boes and Hager 2003), the uniform depth-averaged air concentration was compared with the mean air concentration for uniform flow 

$$C_u = 1 - \frac{h_{w,u}}{h_{90,u}}$$

(2)

as proposed by Hager (1991) for smooth chutes. In order that an experimental $h_{w,u}$-value be retained for the calculation of friction factors, the deviation between $C_u$ and according to Hager (1991) had to be within an arbitrarily selected 20% that is considered sufficiently exact for highly turbulent air-water flow. A total of nine experimental results had to be excluded for friction factor analysis by applying this criterion.

**Flow Depths**

If the uniform equivalent clear water depths are normalized with the step height $s$ and plotted as a function of the roughness Froude number $F_u = q_w/((g \sin \theta) s)^{1/2}$, with $q_w$ as water discharge per unit width and $g$ as acceleration of gravity, a data fit yields (Fig. 7)

$$\frac{h_{w,u}}{s} = 0.23 F_u^{0.65}$$

(3)

In the range of the experiments, Eq. (3) may be approximated as

$$\frac{h_{w,u}}{h_c} = 0.215(F_u)^{2/3}$$

(7)

The uniform clear water depth $h_{w,u}$ relative to critical depth $h_c$ thus varies exclusively with the chute angle $\phi$, independent from $s$ and $q_w$.

The uniform mixture depth $h_{90,u}$ determining sidewall height (Boes and Minor 2000) can be described by (Fig. 8)

$$\frac{h_{90,u}}{s} = 0.50 F_u^{0.1 \tan \phi + 0.5}$$

(5)

Therefore, for a certain relative discharge $h_c/s$, both $h_{w,u}$ and $h_{90,u}$ decrease with increasing chute angle.

**Attainment of Uniform Flow**

The length needed for uniform flow to be attained is difficult to determine experimentally because both the uniform equivalent
clear water and the uniform mixture depths are approached asymptotically. A slight deviation in flow depth may thus result in a large error in the drawdown length $x_d$ from the overflow crest to close uniform equivalent clear water flow. However, as demonstrated by Hager and Boes (2000), the drawdown length can be deduced analytically from the general equation for backwater and drawdown curves (Chow 1959)

$$\frac{dh}{dx} = \frac{\sin \phi - S_f}{\cos \phi - F^2},$$

where $h=$local flow depth; $x=$streamwise coordinate; $S_f=$friction slope; and $F= u/(gh)^{1/2}=$local Froude number. Except close to the spillway crest, stepped spillway flow is hypercritical, i.e., $F^2 > 10$, and the hydrostatic term $\cos \phi$ may be neglected compared to the hydrodynamic term $F^2$. Accordingly, Eq. (6) may be simplified as (Hager and Blaser 1998)

$$\frac{dY}{d\chi} = -\sigma (Y - 1) Y$$

with $Y = h/h_c$, $\chi = x/x_c$ where $x_c = h_c^2/(h_{w,u}^2 \sin \phi)$=scaling length; and $\sigma = 10/3$ for turbulent rough flow, based on the Gauckler–Manning–Strickler (GMS) formula. The general solution of Eq. (7) subject to the boundary (subscript 0) condition $Y(\chi = 0) = Y_0$ is

$$\ln \left( \frac{Y - 1}{Y_0 - 1} \right) = -\sigma \chi$$

Because uniform flow is an asymptotic process, and the super-critical flow over macro roughness has a turbulent surface, uniform flow is considered to be attained if the flow depth is within $\pm 2\%$ of $h_{w,u}$. Introducing $Y = 1.02$ and $Y_0 = Y_c = h_c/h_{w,u}$ in Eq. (8), the corresponding distance obtains

$$-\sigma \chi = \ln \left( \frac{0.02}{1.02} \frac{Y_c}{Y_c - 1} \right)$$

For typical values $Y_c > 2.5$, this may be simplified as

$$-\sigma \chi = 3.93 - \frac{Y_c}{Y_c - 1}$$

or with $H_{\text{dam},u}$ as vertical spillway height (Fig. 1), basically as demonstrated by Hager and Boes (2000)

$$\frac{H_{\text{dam},u}}{h_c} \approx \sin \phi \frac{x_a}{h_c} \left[ 1.18 \left( \frac{h_{w,u}}{h_c} \right)^{2/3} \left( \frac{h_c}{h_{w,u}} \right)^{2/3} \right]$$

The relative vertical length $H_{\text{dam},u}/h_c$ to attain uniform flow thus increases significantly with the ratio of critical to uniform depths. Substitution of Eq. (4) into Eq. (11) yields

$$\frac{H_{\text{dam},u}}{h_c} \approx 25.52 \left[ 1 - 0.055(\sin \phi)^{-1/3} (\sin \phi)^{2/3} \right]$$

or when approximated as a power formula

$$\frac{H_{\text{dam},u}}{h_c} \approx 24(\sin \phi)^{2/3}$$

The normalized vertical drawdown length thus increases almost linearly with the chute inclination.

For $\phi \approx 52^\circ$ as is typical for gravity dams, a minimum relative dam height of $H_{\text{dam},u}/h_c = 20.5$ is required according to Eq. (13) to attain uniform flow. This value agrees with the results of Yildiz and Kas (1998), but is smaller than $H_{\text{dam},u}/h_c = 25 - 30$ suggested by Matos and Quintela (1995) and Matos (2000a), or $H_{\text{dam},u}/h_c = 28$ found by Ohtsu et al. (2000) for similar downstream slopes. The difference to the latter can be explained by the previous assumption of being within $2\%$ of the asymptotic uniform depth. Due to the asymptotic behavior of drawdown curves, the distance required is strongly dependent on the assumption of $Y$. The closer $Y$ is to unity, the larger the value for $x_d$ or $H_{\text{dam},u}$. For example, reducing the margin to $0.5\%$ or $Y = 1.005$ results in a relative vertical distance $H_{\text{dam},u}/h_c = 28.1$ for $\phi = 52^\circ$. However, Eq. (13) is considered sufficient for engineering purposes, especially for the calculation of residual energy at the spillway toe based on equivalent clear water depths.

**Friction Factor**

**Basic Equation**

To account for flow aeration, the Darcy–Weisbach friction factor $f_w$ in uniform two-phase flow was computed with the uniform equivalent clear water depth $h_{w,u}$. With $\sin \phi = r/(2b)[f_w/D_{h_{w,u}}]$, where the hydraulic diameter for uniform flow is $D_{h_{w,u}} = 4h_{w,u} = 4h_{w,u}^2/(b + 2h_{w,u})$, $R_{h_{w,u}}$ being the hydraulic radius, and $u = u_{h_{w,u}}$, the flow velocity computed from the continuity equation, one obtains

$$f_w = \frac{8gbh_{w,u}^3 \sin \phi}{q_w^2 (b + 2h_{w,u})}$$

**Shape Correction Factor**

To account for the rectangular cross-sectional channel shape that departs from the circular shape underlying the Prandtl–Colebrook equation, a shape correction factor $w$ originally introduced by Marchi (1961) was used for the effective hydraulic diameter $D_{h,\text{eff}} = w \cdot D_{h,w}$ as described by Schröder (1990)

$$w = 0.90 - 0.38 \exp \left( \frac{-5h}{b} \right) \quad \text{for } h/b \geq 0.4 \quad (15a)$$

$$w = 0.60 \quad \text{for } h/b < 0.4 \quad (15b)$$

The equivalent clear water depth $h_{w,u}$ was considered as the relevant flow depth $h$ in Eq. (15).

For uniform skimming flow over steps of roughness height $K = s \cdot \cos \phi$ perpendicular to the pseudobottom the Prandtl–Colebrook equation thus reads

$$\frac{1}{\sqrt{f_w}} = -2 \log \left( \frac{2.51}{wR\sqrt{f_w}} + \frac{K/(wD_{h,w})}{3.71} \right)$$
where \( \mathcal{R} = uD_{b,w}/\nu = 4Q_w/[\nu(b + 2h_w)] \) is the Reynolds number, with \( \nu \) = kinematic viscosity of water; and \( Q_w \) = water discharge.

**Sidewall Correction Method**

Because only the channel bottom is covered with the step macro roughness, whereas the sidewalls are hydraulically smooth, the friction factor computed from Eq. (14) accounts for the overall friction behavior of the sectional stepped channel. To determine the friction factor \( f_b \) of the bottom roughness only, the sidewall correction method described by Schroeder (1990) was applied, consisting of the following procedure:

1. Value of \( f_s \) was calculated from Eq. (14).
2. Sidewall skin friction factor \( f_s \) for hydraulically smooth flow was determined by iteratively solving
   \[
   \frac{1}{\sqrt{f_s}} = -2 \log \left[ \frac{2.5f_w}{wR\sqrt{f_b}} \right]
   \]
   with \( w \) from Eq. (15) and \( \mathcal{R} = 4Q_w/[\nu(b + 2h_{w,w})] \).
3. Bottom friction factor \( f_b \) was then obtained from \( f_b = f_w + 2(f_w - f_s)h_{w,w}/b \).

The resulting bottom friction factor \( f_b \) is thus slightly larger than the overall Darcy–Weisbach friction factor \( f_w \) as calculated from Eq. (14). Fig. 9 shows all \( f_b^{1/2} \)-values of the present study as well as those of Yasuda and Ohtsu (1999) for \( \phi = 19^\circ \) as a function of relative roughness \( K/D_{b,eff} \). The data of Yasuda and Ohtsu (1999) were recalculated using the previous sidewall correction method.

**Effect of Roughness Spacing**

Fig. 9 shows an effect of chute slope on the friction factor except for \( \phi = 40 \) and 50° with about equal \( f_b \) values. This may be explained by a different distance \( L_s = s/\sin \phi = K(\sin \phi \cos \phi) \) between two adjacent step corners for a given step roughness \( K \) (Fig. 1) except for \( \phi = 40 \) and 50° with equal \( L_s = K/\sin(2\phi) \). Since \( L_s \) determines the shear length between the recirculating vortices in the step niches and the main flow along the pseudo-bottom, \( f_b \) is, e.g., larger for \( \phi = 30^\circ \) with \( K/L_s = \sin(2\phi)/2 \) = 0.433 compared to \( \sin(2\phi)/2 = 0.492 \) for 40 and 50°. This observation agrees with those of Yasuda and Ohtsu (1999). Friction factors of \( f_b \approx 0.11 \) for \( \phi = 30^\circ \) and \( f_b \approx 0.07 \) for \( \phi = 50^\circ \) were obtained in the present study, whereas those of Yasuda and Ohtsu (1999) for 30° and 55° are \( f_w = 0.17 \) and 0.14, respectively.

**Effect of Depth Measurement**

Yasuda and Ohtsu (1999) used a nonintrusive method to determine the equivalent clear water depth by measuring the seepage depth of a hydraulic jump at the spillway toe. This apparently results in an overestimation of the uniform clear water depth

\[
h_w = \int_0^{h_{w,0}} [1 - C(y)] dy
\]

as determined in the present study, and thus leads to larger friction factors according to Eq. (14) in the study of Yasuda and Ohtsu (1999) for \( \phi = 30^\circ \) and 55°. In Eq. (18), \( C(y) \) is the local air concentration at an outer step edge cross section, and \( y \) is the coordinate normal to the pseudo-bottom. However, because the flow aeration is small for \( \phi = 19^\circ \), deviations from friction factors obtained with direct depth measurement are believed to be negligible for this small chute angle, and the corresponding data of Yasuda and Ohtsu (1999) are plotted in Fig. 10 for the sake of comparison.

**General Determination of Friction Factor**

As can be seen in Fig. 9, the friction factor of skimming flow over stepped spillways is basically a function of relative roughness \( K/D_{b,w} \) and roughness spacing \( K/L_s = \sin(2\phi)/2 \)

\[
f_b = \Pi_1(K/L_s) \cdot \Pi_2(K/D_{b,w})
\]

Using a function \( f_b(K/L_s = \text{const}) = \Pi_2(K/D_{b,w})^{0.2} \) for a given roughness spacing, the effect of \( K/L_s = \sin(2\phi)/2 \) was fitted as \( \Pi_1 = 0.5 - 0.42 \sin(2\phi) \). All data of the present study and those of Yasuda and Ohtsu (1999) for \( \phi = 19^\circ \) thus fall on the curve (Fig. 10)

\[
f_b = [0.5 - 0.42 \sin(2\phi)]^{0.2}
\]

Approximations and \( f_w \) data, respectively, of Tozzi (1992) for \( \phi = 26.6^\circ \), curve [1], and for \( \phi = 53.1^\circ \), curve [2], of Frizell et al. (1994) for \( \phi = 26.6^\circ \), curve [3], of Wahrheit-Lensing (1996) for \( \phi = 51.3^\circ \), (\( \bigcirc \)) Yasuda and Ohtsu (1999) for \( \phi = 19^\circ \), ([4], — — —) Chamani and Rajaratnam (1999b) for \( \phi = 53.1^\circ \), Chanson et al. (2000) for \( \phi = [5], — — — 19^\circ \) based on model data, and ([6], — — —) 51.3°, present study for \( \phi = (\bigcirc) 30^\circ \) and (■) 40°/50°, (—) Eq. (20).

Fig. 9. Bottom friction factor \( f_b^{1/2} \) as function of relative roughness \( K/D_{b,eff} \) for (\( \bigcirc \)) \( \phi = 19^\circ \) (Yasuda and Ohtsu 1999), present study for \( \phi = (\bigcirc) 30^\circ \), (■) 40°, and (\( \bigtriangleup \)) 50°

Fig. 10. Friction factor \( f_w \) and bottom friction factor \( f_b \), respectively, as functions of \( \Pi_1 \cdot \Pi_2 \) with experimental data or empirical equations of Tozzi (1992) for \( \phi = ([1], — — — 26.6^\circ \) and ([2], — — —) 53.1°; ([3], — — —) Frizell et al. (1994) for \( \phi = 26.6^\circ \), (\( \bigtriangleup \)) Wahrheit-Lensing (1996) for \( \phi = 51.3^\circ \), (\( \bigcirc \)) Yasuda and Ohtsu (1999) for \( \phi = 19^\circ \), ([4], — — —) Chamani and Rajaratnam (1999b) for \( \phi = 53.1^\circ \), Chanson et al. (2000) for \( \phi = [5], — — — 19^\circ \) based on model data, and ([6], — — —) 51.3°, present study for \( \phi = (\bigcirc) 30^\circ \) and (■) 40°/50°, (—) Eq. (20)
for \(\phi=51.3^\circ\), curve [6], were also added to Fig. 10. The friction factors \(f_m\) generally represent the overall friction behavior except for the authors’ experimental data and those of Yasuda and Ohtsu (1999) which stand for \(f_b\) values. The present prediction Eq. (20) is surrounded by the various curves [1]–[6] and by the data of Wahrheit-Lensing (1996). This demonstrates the difficulties to determine the friction factor in a highly turbulent two-phase flow across a stepped spillway.

Provided \(0.1<K/D_{h,w}<1.0\), Eq. (20) may also be expressed by a familiar logarithmic function. The bottom roughness friction factor \(f_b\) on stepped chutes with \(19^\circ<\phi<55^\circ\) as given by Eq. (20) can be approximated by

\[
\frac{1}{\sqrt{f_b}} = \frac{1}{0.5 - 0.42 \sin(2\phi)} \left[ 1.0 - 0.25 \log \left( \frac{K}{D_{h,w}} \right) \right]^{4} \tag{21}
\]

Both Eqs. (20) and (21) demonstrate that the effect of chute angle \(\phi\) is much larger than that of relative roughness. This important observation is also reflected by Eq. (4), describing the uniform equivalent clear water depth.

**Effect of Aeration on Friction Factor**

The presence of air within turbulent boundary layers reduces the shear stress. The resulting drag reduction leads to a decrease of energy dissipation on chutes (Chanson 1994). Former model studies normally overestimated the friction factor and the energy dissipation rate due to insufficient consideration of aeration. Often, flow depths in two-phase flow were simply measured with point gages or scales attached to the channel sidewalls. These measurements resulted in flow depths that rather described the characteristic mixture depth \(h_{90}\) instead of the clear water depth \(h_w\), based on knowledge of the air concentration according to Eq. (18). If the friction factor was subsequently calculated from Eq. (14) with \(h=h_{90,a}\) instead of \(h=h_{w,a}\), air entrainment was completely neglected, resulting in a friction factor \(f_m\) of pseudouniform flow depth \(h_{90,a}\).

The significant reduction of the friction factor due to air entrainment along a spillway is shown in Fig. 11 where the ratio \(f_w/f_m\) is plotted. All experimental data, regardless of the chute slope, fall on a line given by (Boes 2000)

\[
f_w/f_m = 0.5 \left(1 + \tanh \left( \frac{0.25 - \bar{C}}{\bar{C}(1-\bar{C})} \right) \right) \tag{22}
\]

This approximation resembles that for drag reduction on smooth chutes due to air entrainment by Chanson (1994), who considered the friction factor of non-aerated flow \(f\) instead of \(f_m\), however (Fig. 11).

Also plotted in Fig. 11 are the corresponding data of Wahrheit-Lensing (1996) that approach the experimental results of the present study with increasing mean air concentration \(\bar{C}\). The deviation is supposed to result from an overestimation of the characteristic mixture depths measured with point gages. The measured flow depths rather represent \(h_m\) values with \(m > 90\), e.g., \(h_{90}=h(C=0.95)\), instead of \(h_{90}\) as considered in the present study.

**Energy Dissipation**

Knowledge of the residual kinetic energy of a flow at the toe of a spillway is important to design the energy dissipator downstream of a stepped chute. The residual head at any section along a stepped spillway, regardless of uniform or nonuniform flow conditions, can be expressed by

\[
H_{res} = h_w \cos \phi + \alpha \frac{q_w^2}{2g h_w^2} \tag{23}
\]

where \(h_w\) is obtained from Eq. (18) and the energy correction coefficient is \(\alpha \approx 1.1\). All experimental data for \(H_{res}\) were normalized with the maximum reservoir head \(H_{max} = H_{dam} + 1.5h_e\) (Fig. 1). If the normalized residual heads for all spillway chutes tested are plotted over a normalized dam height (Fig. 12), no distinction can be observed between energy heads for \(\phi = 40^\circ\) and \(50^\circ\), supporting the previously discussed equal behavior of friction factors for equal roughness spacing \(K/L_e\). Also, for a given relative dam height, the energy heads are smallest for \(19^\circ\) because of higher \(f_b\)-values compared to \(30, 40,\) and \(50^\circ\).

For the calculation of residual energy heads on stepped chutes, a distinction should be made between conditions where uniform flow is attained or not. For uniform flow, i.e. for \(H_{dam}/h_e \geq 15\) to 20 according to Eq. (13), Chanson’s approach (1994) was modified by accounting for the boundary condition

\[
H_{res}/H_{max}(H_{dam}/h_e=0) = 1 \tag{12}
\]

For non-uniform spillway flow the data were fitted with an exponential function to result in a normalized residual head.
\[
\frac{H_{\text{res}}}{H_{\text{max}}} = \exp \left[ -0.045 \left( \frac{K}{D_{h,w}} \right)^{0.1} \frac{\sin(\phi) - 0.8}{H_{\text{dam}}/h_c} \right]
\]
\[
\text{for } H_{\text{dam}}/h_c < 15 - 20 \quad (24a)
\]
\[
\frac{H_{\text{res}}}{H_{\text{max}}} = \frac{F}{h_c + F}
\]
\[
\text{for } H_{\text{dam}}/h_c \geq 15 - 20 \quad (24b)
\]
with
\[
F = \left( \frac{f_b}{8 \sin(\phi)} \right)^{1/3} \cos(\phi) + \frac{\alpha}{2} \left( \frac{f_b}{8 \sin(\phi)} \right)^{-2/3}
\]

The hydraulic diameter \(D_{h,w}\) in Eq. (24a) should be computed with \(h_{w,u}\) from Eq. (4), and the friction factor \(f_b\) in Eq. (24b) from Eq. (20) or (21). It should be noted that a direct computation of residual energy head, based on either a drawdown curve, or on Eq. (4) for uniform mixture flow, may be simpler. Yet, Fig. 12 gives an idea of the main parameters involved and Eq. (24) was retained, therefore.

Training Wall Design
The considerable aeration on stepped chutes leads to flow bulking which should be accounted for in the design of spillway training walls. According to Boes and Minor (2002), the proposed sidewall design height \(h_d\) reads
\[
h_d = \eta \cdot h_{90,u}
\]
with a safety factor \(\eta = 1.2\) for concrete dams without concern for erosion on the downstream face, and \(\eta = 1.5\) in case of emergency spillways on embankment dams prone to erosion. The safety factor takes into account the relatively larger spray height in the prototype due to a higher turbulence degree, as compared to the model results (Boes and Minor 2000). It should be noted, however, that Eq. (25) is based upon the skimming flow regime without spillway aerator. In case of nappe flow, the nappe impact on the steps may cause a considerable spray that might overtop the training walls designed after Eq. (25) (Boes and Minor 2002; see also Design Example).

Design Example
Assume that a stepped chute is to be designed for a dam with the following boundary conditions: \(H_{\text{dam}} = 60\) m vertical dam height above stilling basin; \(b = 40\) m downstream river width; \(1V:0.8H\) slope of downstream dam face; \(\phi = 51.3^\circ = \arctan(1/0.8)\) spillway chute angle; \(Q_d = 800\) m\(^3\)/s design discharge; and RCC lift thickness 0.6 m.

Selection of Spillway Width
To avoid converging spillway training walls, which lead to the creation of shock waves, a chute width equal to the downstream river width is chosen, i.e., \(b = 40\) m. This results in a unit discharge of \(q_d = Q_d/b = 800/40 = 20\) m\(^3\)/s (m). For a rectangular spillway cross section, the critical depth is \(h_c = (q_d^2/g)^{1/3} = (20^2/9.81)^{1/3} = 3.44\) m.

Selection of Step Height and Flow Regime
On account of the given RCC lift thickness, a step height of \(s = 1.2\) m is chosen, facilitating spillway construction on the one hand, and ensuring a large energy dissipation rate on the other. For the given design discharge, the ratio \(h_{c}/s = 3.44/1.2 = 2.87\) is more than 20% beyond the value of \(h_{c}/s = 0.74\) given by Eq. (1) for the onset of skimming flow. In other words, nappe flow takes place only for small discharges up to about \(q_w = (h_{c}/s)^{1/2}\)

\[
= (0.74 \times 1.2)^{1/2}\) = 2.6 m\(^3\)/s(m), \) when skimming flow sets in.

Inception Point of Air Entrainment
Eq. (4) in Boes and Hager (2003) indicates a distance of \(L_i = 55.4\) m between the spillway crest and the inception point location (see Fig. 1), i.e. the white water reach starts about halfway along the chute.

Inception Flow Depth
The flow depth at the inception point amounts to \(h_{m,i} = 1.33\) m according to Eq. (5) from Boes and Hager (2003). The two-phase flow velocity is thus \(v_{w,i} = q_d/h_{w,i} = 20/1.33\) m/s. Because the depth-averaged inception air concentration \(C_{\bar{u}} = 0.23\) from Eq. (7) (Boes and Hager 2003), the inception clear water depth is only about \(h_{w,i} = h_{m,i}(1 - C_{\bar{u}}) = 1.33(1 - 0.23) = 1.02\) m, resulting in an inception clear water velocity of about \(v_{w,i}\) = 20/1.02 = 19.6 m/s. This value is just below the critical velocity of roughly 20 m/s for the inception of cavitation in unaerated stepped chute flow (Boes and Hager 2003).

Attainment of Uniform Flow
According to Eq. (13), the vertical distance required for uniform flow to be attained would be about \(H_{\text{dam},u} = 70\) m, which is more than the spillway height. The flow is therefore still developing at the chute end, but not far from uniform conditions.

Uniform Flow Depths
If the spillway was sufficiently long for uniform flow to be established, the uniform equivalent clear water depth would be \(h_{w,u} = 0.80\) m from Eq. (4), whereas the uniform characteristic mixture depth would amount to \(h_{90,u} = 1.74\) m from Eq. (5), with \(F_{s} = 20/(9.81 \times \sin(51.3^\circ)) = 2.87\) is \(0.74\) given by Eq. (2). The uniform depth-averaged air concentration would thus be \(C_{\bar{u}} = 0.54\) from Eq. (2).

Energy Dissipation
Because uniform flow is not attained, the energy dissipation is computed from Eq. (24a). The required equivalent clear water depth at the chute end \(h_{w,e}\) can be approximated by linear interpolation between the inception depth \(h_{w,i} = 1.02\) m and the uniform depth \(h_{w,u} = 0.80\) m at vertical distances from the crest of \(z_i = L_i \sin(\phi) = 27.7\) m and \(H_{\text{dam},u} = 70\) m, respectively: \(h_{w,e} = h_{w,i} - (h_{w,i} - h_{w,u})/(H_{\text{dam},u} - z_i) = (H_{\text{dam},u} - z_i) = 0.85\) m. With \(D_{h,w} = 4h_{w} = 4 \times 0.85 = 3.4\) m and \(K = s \cdot \cos(\phi) = 1.2 \cdot \cos(51.3^\circ) = 0.75\) m the relative roughness is \(K/D_{h,w} = 0.75/3.4 = 0.22\). The residual energy head \(H_{\text{res}}\) at the chute end is thus \(28.6\) m from Eq. (24a) compared with a maximum head of \(H_{\text{max}} = H_{\text{dam}} + 1.5h_{c} = 65.16\) m in the reservoir, with both heads referring to the stilling basin bottom at the dam toe (Fig. 1). About \(1 - 28.6/65.16 = 56\%\) of the kinetic energy have thus been dissipated along the chute.

Solving Eq. (23) for \(h_{w}\) results in an equivalent clear water depth at the chute end of \(h_{w,e} = 0.89\) m, corroborating the interpolated value of 0.85 m. Finally, inserting the mean value \(h_{w,e}\)
\[(h_w,s) + h_w,e)/2 = 0.87 \text{ m}\] in the continuity equation yields a terminal velocity of \(v_{w,e} = q_d/h_w,e = 20/0.87 = 23 \text{ m/s}\).

If the chute was long enough for the attainment of uniform flow, i.e., \(H_{\text{dam}} = H_{\text{dam,u}} = 70 \text{ m}\), the normalized residual head would read \(H_{\text{res}}/H_{\text{max}} = 0.36\) according to Eq. (24b), with \(f_s = 0.067\) from Eq. (21), \(D_{h,w} = 4 \cdot 0.80 = 3.20 \text{ m}\) and 0.1 \(< kD_{h,w} = 0.23 < 1.0\). In this case, 64\% of the flow energy of \(H_{\text{max}} = 75.2 \text{ m}\) would be dissipated on the spillway, and the terminal velocity would amount to \(v_{w,e} = 20/0.80 = 25 \text{ m/s}\).

**Training Wall Design**

With \(\eta = 1.2\) for concrete dams, the required sidewall height from Eq. (25) is \(h_{r,0} = 2.09 \text{ m}\), with \(h_{0,0,0} = 1.74 \text{ m}\) from Eq. (5). A sidewall height of 2.1 m is proposed. If the downstream dam face were prone to erosion, and if it were essential to avoid overtopping of the training walls, distinction should be made about whether the crest profile above the point of tangency is smooth or stepped. In the latter case, the required wall height should be at least \(h_{r,0} = 1.5h_{0,0,0} = 1.5 \text{ m}\) or for a smooth crest profile, the wall height should be \(h_{r,0} = h_{r,0,0} = 4 \times 2.1 = 4.8 \text{ m}\) over about \(L = 25s = 25 \times 1.2 = 30 \text{ m}\) from the crest to allow for the spray resulting from nappe impact on the first steps below the smooth crest (Boes and Minor 2002).

**Conclusions**

The following findings of the present experimental study apply:

1. The onset of skimming flow is expressed by the ratio of critical depth to step height and follows a linear function as expressed in Eq. (1).
2. The uniform equivalent clear water depth \(h_{w,u}\) on stepped spillways depends on the chute angle and unit discharge only, as given in Eq. (4).
3. The characteristic uniform mixture depth \(h_{0,0,0}\), according to Eq. (5) is a function of step height, unit discharge and chute angle.
4. The drawdown length to the approximate location of uniform flow attainment as given in Eq. (13) depends on chute angle and unit discharge only.
5. The bottom roughness friction factor is approximated for a wide range of spillway angles and relative roughness by Eq. (20) or (21).
6. The significant effect of aeration on the reduction of friction factors is illustrated by the ratio \(f_w/f_m\) as function of the mean air concentration, Eq. (22), where \(f_w\) and \(f_m\) are friction factors with and without consideration of flow aeration, respectively.
7. A general expression of residual energy head along stepped chutes is given in Eq. (24), with distinction between developing and uniform flow regions.
8. Stepped spillway training walls should be designed according to Eq. (25), taking into account the erosion potential of the downstream dam face.

These conclusions in conjunction with the results of Boes and Hager (2003) allow for the hydraulic design of stepped spillways for a wide range of boundary conditions including typical applications both for embankment and gravity dams.

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**Notation**

The following symbols are used in this paper:

- \(b\) = spillway or river width;
- \(C\) = depth-averaged air concentration;
- \(C_w\) = depth-averaged air concentration at inception point;
- \(C_{w,u}\) = uniform depth-averaged air concentration;
- \(C(y)\) = local air concentration;
- \(D_{h,w}\) = hydraulic diameter;
- \(D_{h,eff}\) = effective hydraulic diameter;
- \(F\) = uniform equivalent clear water flow;
- \(H_{\text{dam}}\) = vertical spillway or dam height;
- \(H_{\text{dam,u}}\) = vertical distance from spillway crest to close uniform equivalent clear water flow;
- \(H_{\text{res}}\) = maximum reservoir energy head;
- \(h\) = local flow depth;
- \(h_i\) = critical depth;
- \(h_{f}\) = training wall design height;
- \(h_{m}\) = mixture depth;
- \(h_{m,u}\) = mixture depth at inception point;
- \(h_{spray}\) = spray height resulting from nappe impact on steps;
- \(h_{w}\) = uniform flow depth;
- \(h_{w,e}\) = equivalent clear water depth;
- \(h_{w,u}\) = clear water depth at chute end;
- \(h_{0}\) = uniform equivalent clear water depth;
- \(h_{0,0}\) = characteristic mixture depth with local air concentration of \(C = 0.90\);
- \(h_{0,0,u}\) = uniform characteristic mixture depth;
- \(K\) = depth-averaged air concentration of \(K/(\sin \phi \cos \phi) = 2K/(\sin 2\phi)\) distance between step edges, roughness spacing;
- \(L_i\) = black water length from spillway crest to inception point;
- \(L_s\) = flat water length from step edge to inception point;
- \(q_d\) = design discharge;
- \(Q_d\) = design discharge per unit width;
- \(q_m\) = water discharge per unit width;
- \(R\) = Reynolds number;
- \(R_{h,w}\) = hydraulic radius;
- \(S_f\) = friction slope;
- \(s\) = step height;
- \(u\) = flow velocity in \(x\) direction;
- \(v_{m,e}\) = mixture velocity at inception point;
- \(v_{w,e}\) = clear water velocity at chute end;


\[ u_{w,i} = \text{clear water velocity at inception point;} \]
\[ w = \text{shape correction coefficient;} \]
\[ x = \text{streamwise coordinate originating at spillway crest;} \]
\[ x_i = h_i^2/(h_{w,i}^2 \sin \phi) \text{ scaling length;} \]
\[ x_u = \text{drawdown length from spillway crest to close uniform equivalent clear water flow;} \]
\[ Y = h/h_u \text{ normalized local flow depth;} \]
\[ Y_c = h_c/h_u \text{ normalized critical depth;} \]
\[ y = \text{transverse coordinate originating at pseudobottom;} \]
\[ z_i = \text{vertical black water length from spillway crest to inception point;} \]
\[ \alpha = \text{energy correction coefficient;} \]
\[ \eta = \text{safety factor;} \]
\[ \nu = \text{kinematic viscosity of water;} \]
\[ \Pi_1 = 0.5-0.42 \sin(2\phi) \text{ function taking into account roughness spacing;} \]
\[ \Pi_2 = (K/D_{sw})^{0.2} \text{ function taking into account relative chute roughness;} \]
\[ \sigma = \text{factor originating from Gauckler–Manning–Strickler formula;} \]
\[ \phi = \text{chute angle from horizontal;} \]
\[ \chi = x/x_s \text{ normalized streamwise coordinate.} \]

References


